The Statistical Mechanics of Financial Markets

Edoardo Luna

Feel free to interrupt and ask questions at any time
I am assuming no knowledge in finance or stochastic processes.
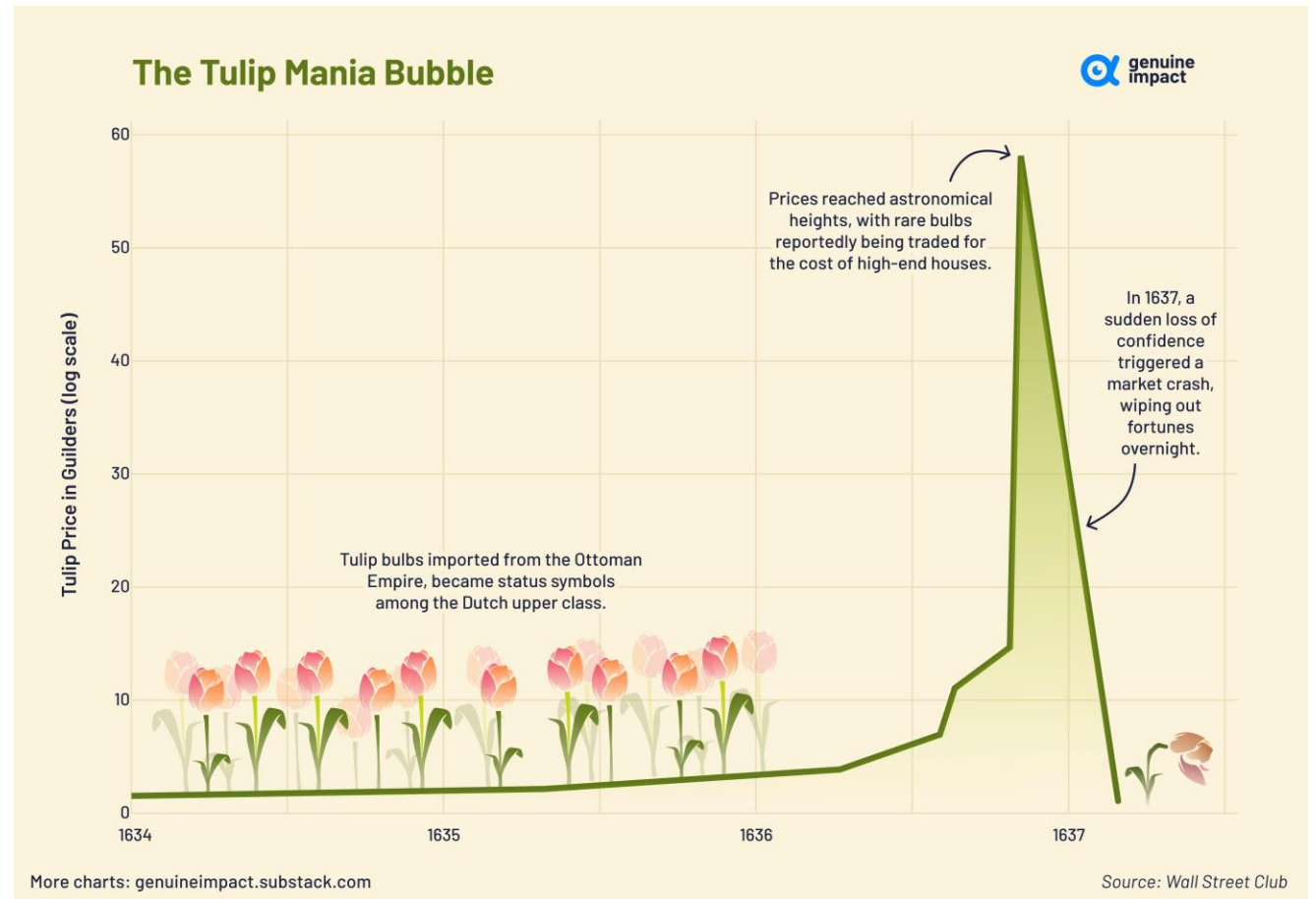Not my research, just a hobby
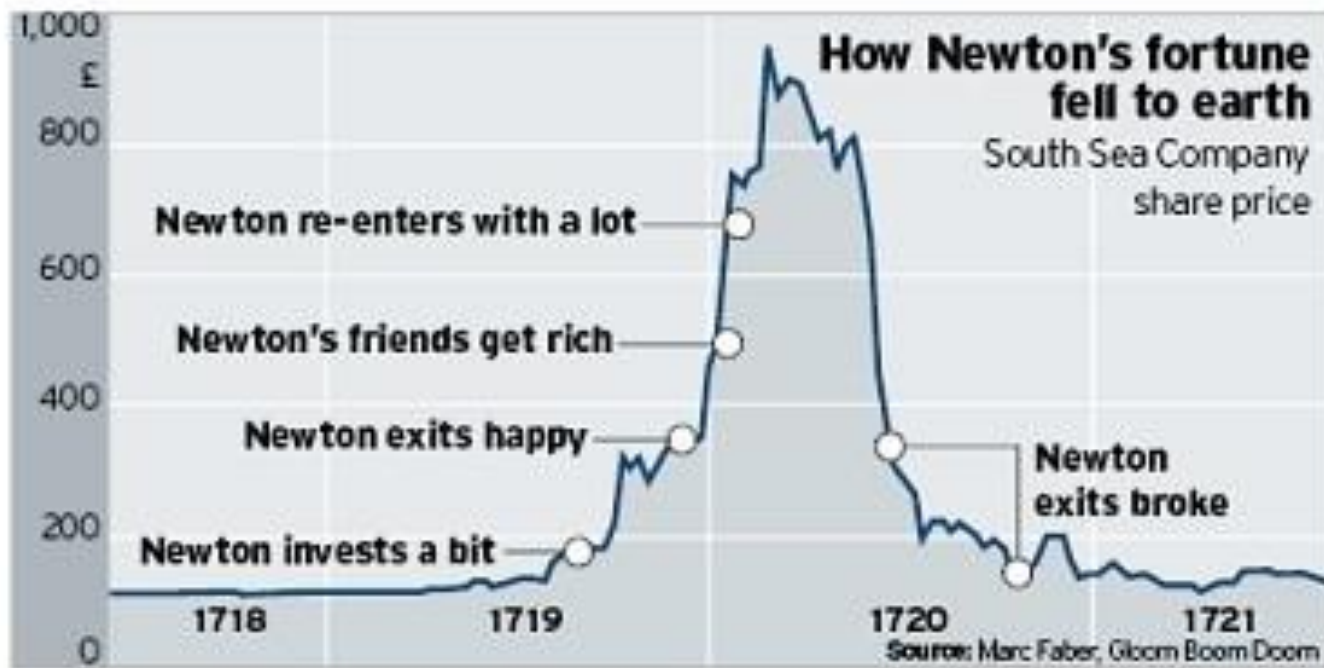
A Brief History of Market Crashes

- Tulip Mania (1585-1650)
- South Sea Bubble (1720 & Isaac Newton)
- October 1929
- October 1987
- Dot-com Bubble 1999-2001
- 2008 Financial Crisis
- Many many more

Tulip Mania in 17th Century Netherlands

Prices detached from true valuation and reality at the peak. Driven by speculation and euphoria

South Sea Bubble (Early 1700s)

How Newton's fortune fell to earth
South Sea Company share price

Newton re-enters with a lot
Newton's friends get rich
Newton exits happy
Newton invests a bit
Newton exits broke

1718   1719   1720   1721
Source: Marc Faber, Gloom Boom Doom

"I can calculate the movement of stars, but not the madness of men"- Newton

Dot-Com Bubble 1999-2001

Not just Cisco. All dot-com companies boomed with many going bust.
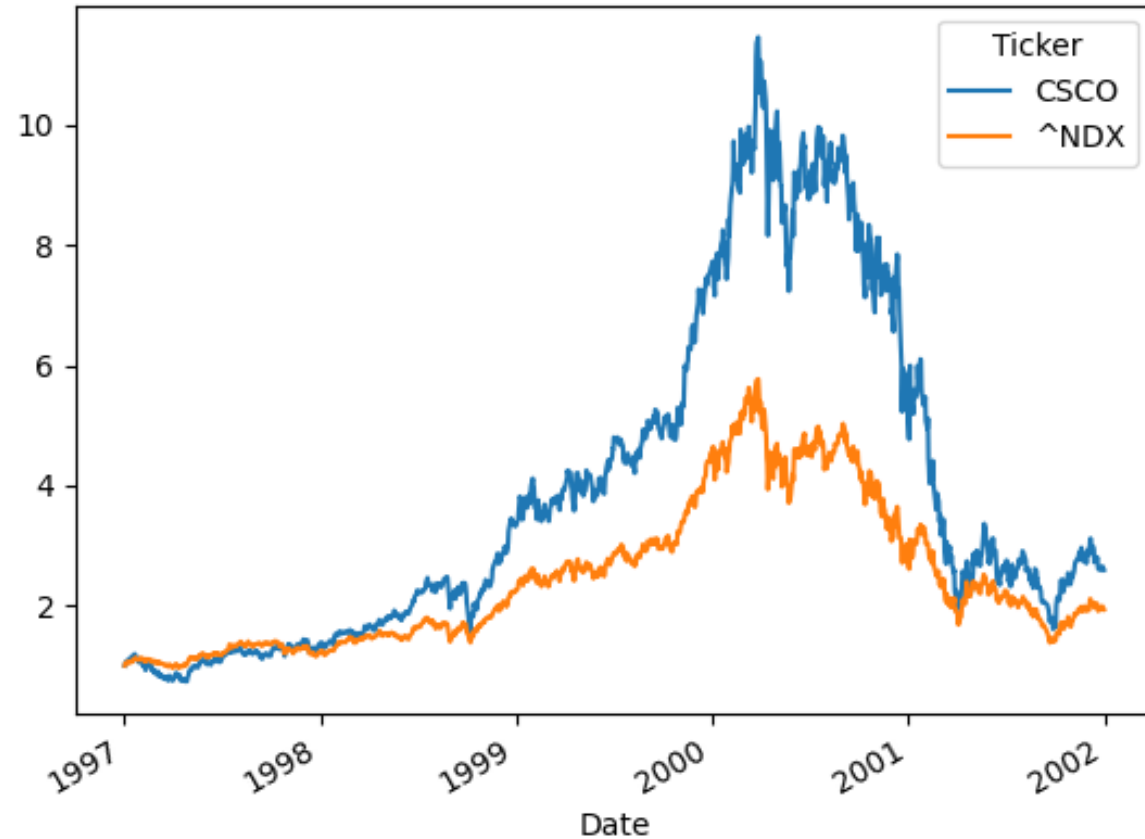
Louis Bachelier & Brownian Motion 1900

- Louis Bachelier published a model for stock prices in Paris 1900, 5 years before Einstein's Brownian motion. Same mathematics ie Random walks and diffusion

- Advised by Henri Poincare

$$p(x,t) = \frac{1}{2}\,p(x + \Delta x, t - \Delta t) + \frac{1}{2}\,p(x - \Delta x, t - \Delta t)$$

$$\frac{1}{2}\Big[p(x + \Delta x, t - \Delta t) + p(x - \Delta x, t - \Delta t)\Big]$$

$$= \frac{1}{2}\Big[2p(x,t) - 2\Delta t\,p_t(x,t) + (\Delta x)^2\,p_{xx}(x,t)\Big]$$

$$= p(x,t) - \Delta t\,p_t(x,t) + \frac{1}{2}(\Delta x)^2\,p_{xx}(x,t).$$

$$p_t(x,t) = \frac{(\Delta x)^2}{2\,\Delta t}\,p_{xx}(x,t). \qquad \text{PDEs from probability}$$

Famous Quants

Jim Simons Math PhD (Renaissance Tech) Net worth ~$30B. Well known for Chern-Simons forms & starting the Flatiron Institute

Ed Thorp Math PhD (TGS Management) Net Worth ~$1B. Invented card counting in blackjack and pioneered options pricing. Worked w/ Claude Shannon
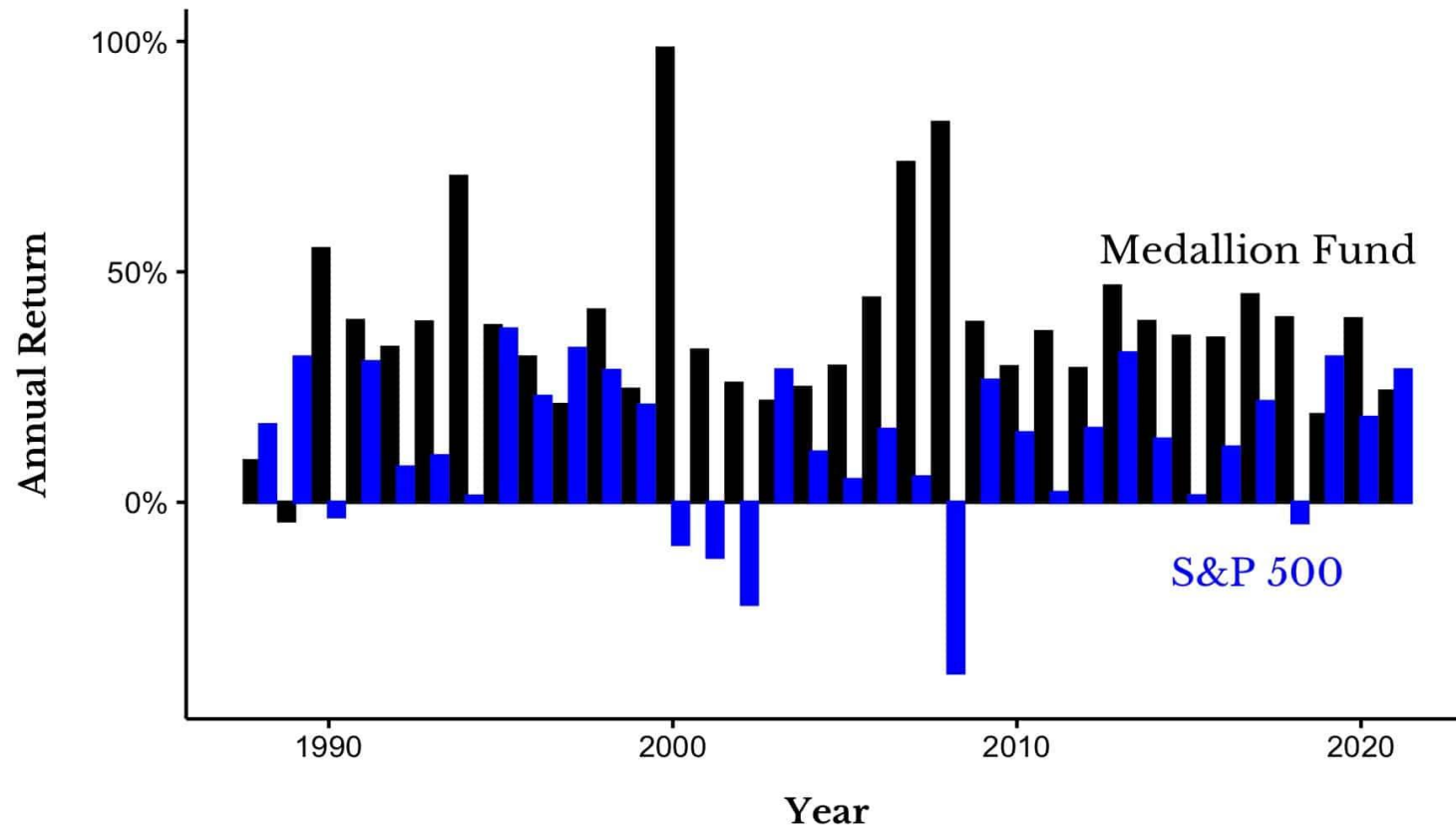

Doyne Farmer & Norman Packard Physics PhDs (Prediction company). Farmer is a physics professor at Oxford. Sold the company to Union Bank of Switzerland for hundreds of millions

Renaissance Annual Returns

Annual Returns for the S&P 500 vs.
The Medallion Fund (Net of Fees)
1988-2021

Source: DFA, Gregory Zuckerman (OfDollarsAndData.com)
Note: Shows the total annual returns for the S&P 500 and the Medallion Fund (net of fees).
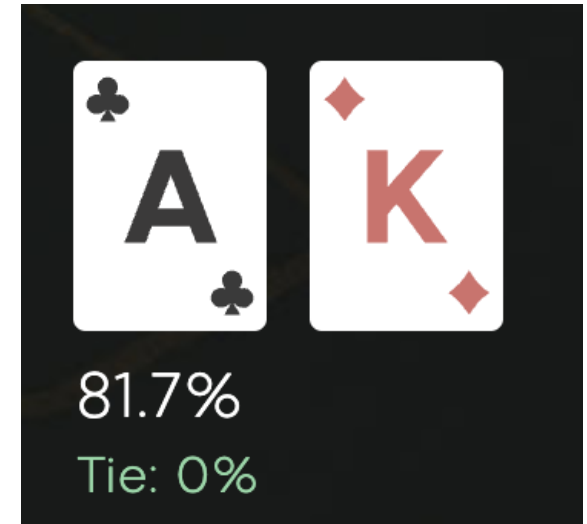
Trading: An analogy with poker
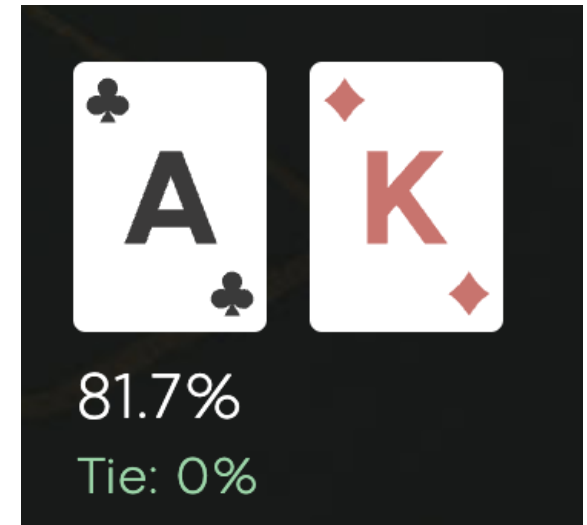
Hero: 100BB

Should you take the bet?

POT: 80BB

Win:
18.3%

A 7 8 Q

81.7%
Tie: 0%

Trading: An analogy with poker

Hero: 100BB

Should you take the bet?

POT: 80BB

Villain:
Pushes All in
with 20BB

Win:
18.3%

81.7%
Tie: 0%

~20%
or 1/5

Yes! Payout is 5:1. You only need 4:1 to have a positive Expected Value

Same thing in finance. Identify profitable opportunities

Memoryless Property of Random Walks

Consider a discrete random walk formed by steps +1/-1
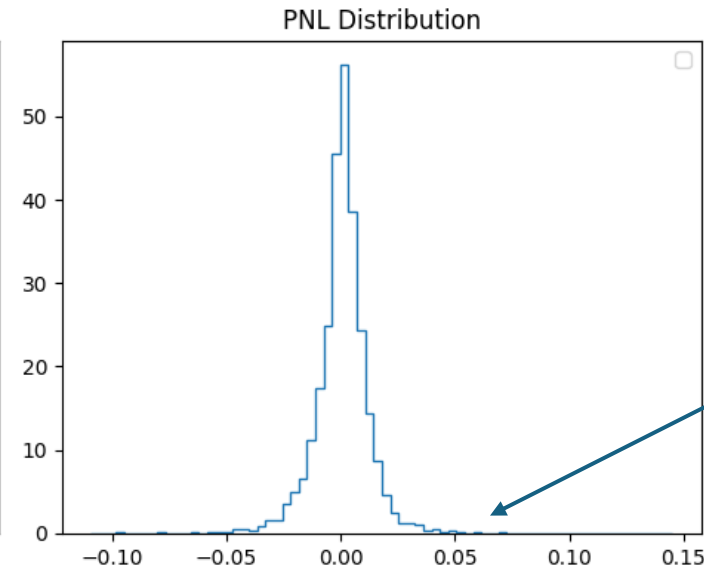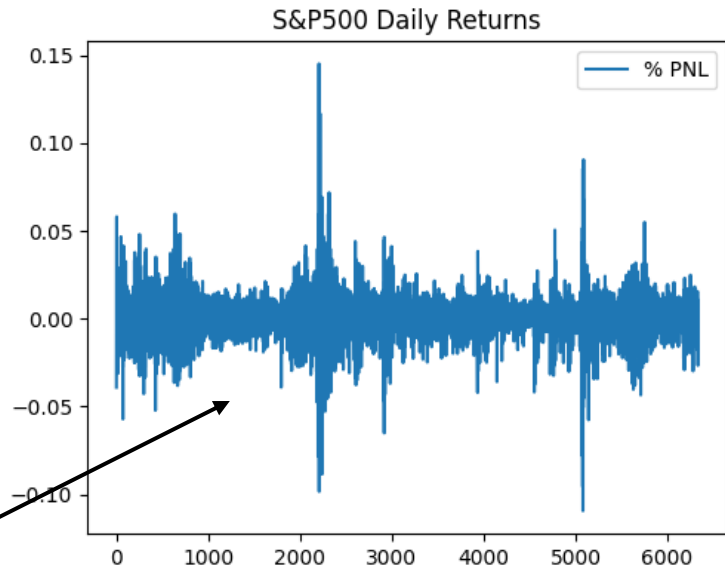
Definition of Drawdowns/ups D: Consecutive ups or downs

$$P(X_i = 1) = p \text{ Probability for step i to be heads}$$

$$P(D = k) = q^{k-1}p \textbf{ Geometric Distribution!}$$

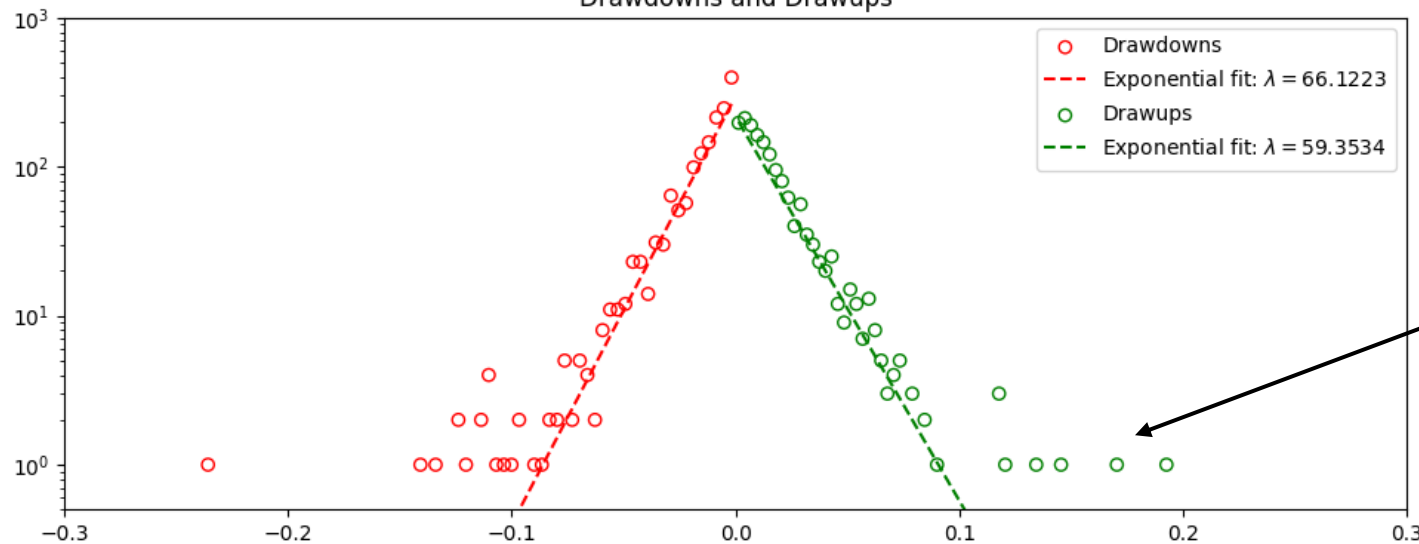$$P(D = k) = q^{k-1}p \rightarrow P(x) = \lambda \exp(-\lambda x) \text{ Continuum limit}$$

The Basics: Statistical Data of S&P500 since 2000

Similarity to velocity fluctuations in turbulence
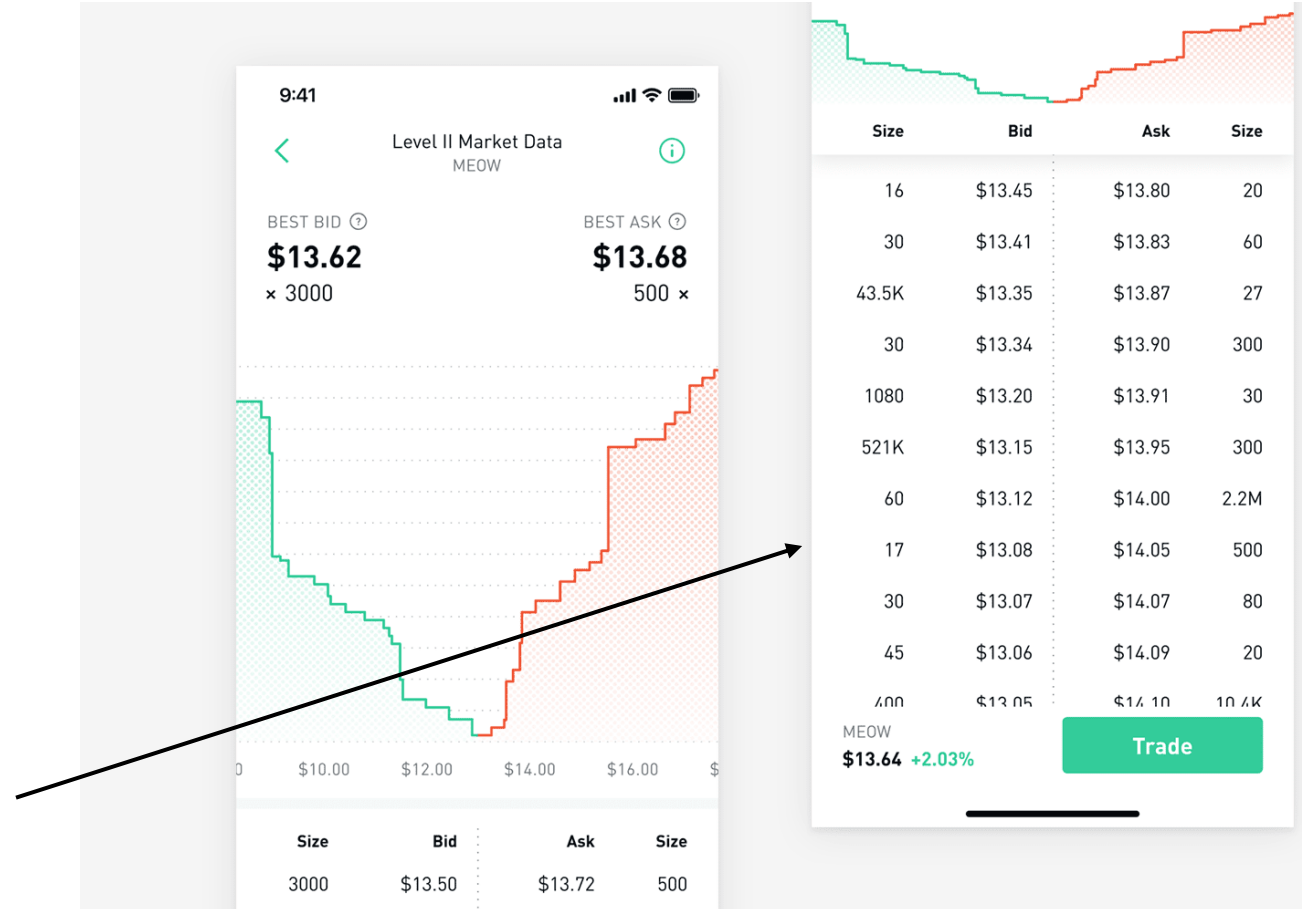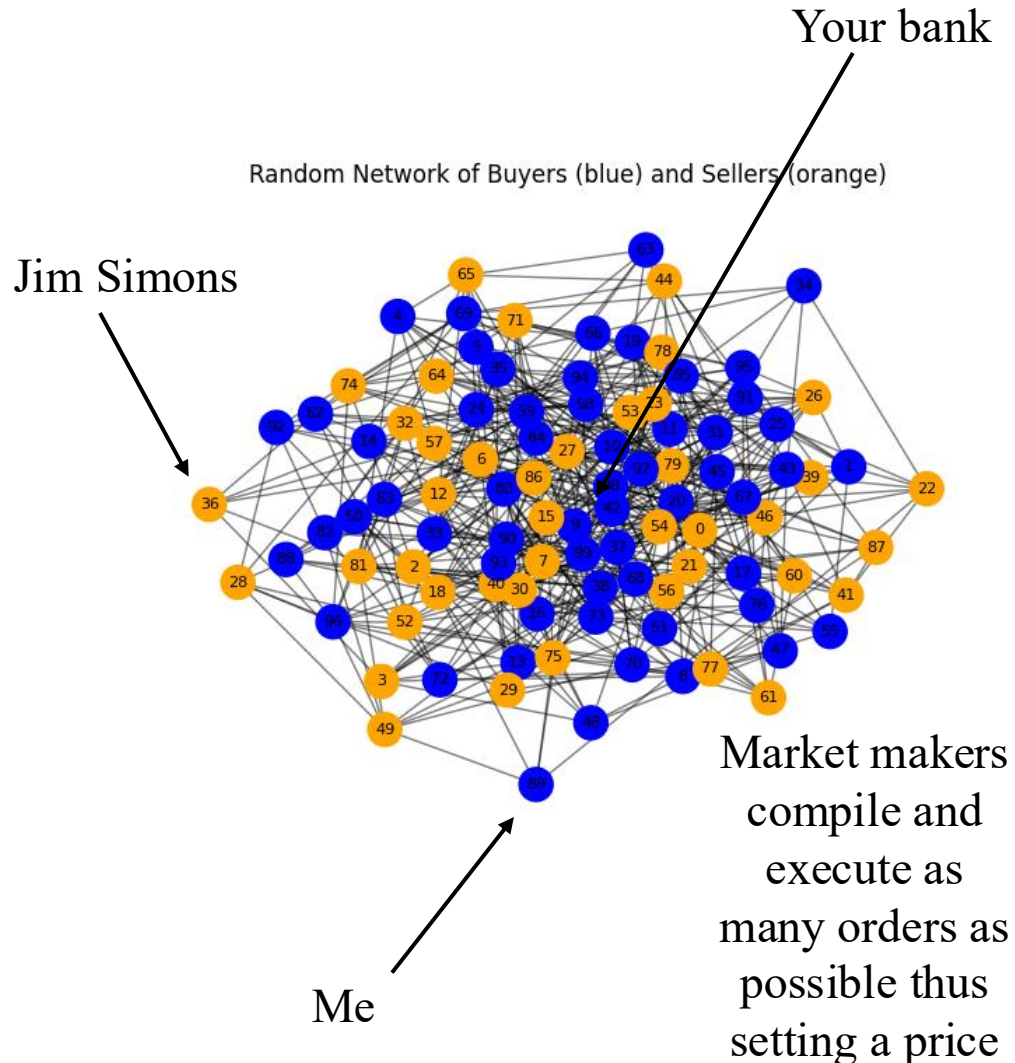
Fat Tails, ie non Gaussian Statistics

Volatility Clustering

Can do this calculation for different scales!

Drawup anomalies

What is a market? Market Microstructure

Random Network of Buyers (blue) and Sellers (orange)

Your bank

Jim Simons

Me

Market makers compile and execute as many orders as possible thus setting a price

Level II Market Data
MEOW

9:41

BEST BID ⑦
$13.62
× 3000

BEST ASK ⑦
$13.68
500 ×

$10.00 $12.00 $14.00 $16.00

Size	Bid	Ask	Size
3000	$13.50	$13.72	500

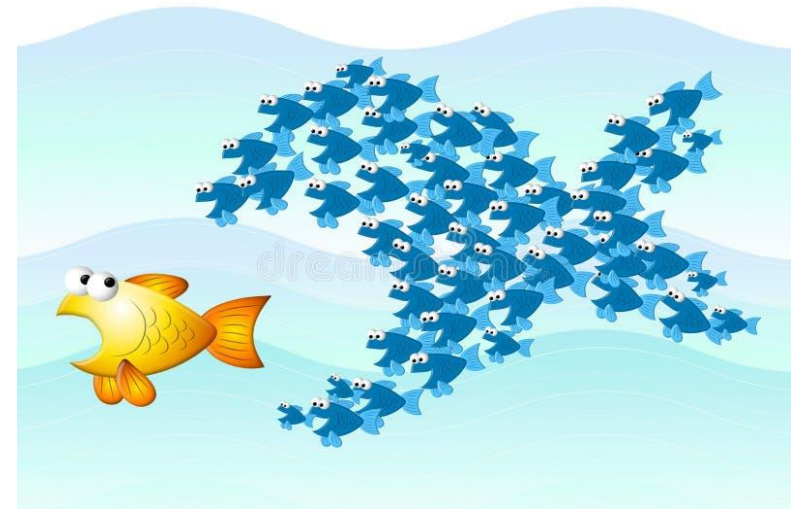| Size	Bid	Ask	Size
16	$13.45	$13.80	20
30	$13.41	$13.83	60
43.5K	$13.35	$13.87	27
30	$13.34	$13.90	300
1080	$13.20	$13.91	30
521K	$13.15	$13.95	300
60	$13.12	$14.00	2.2M
17	$13.08	$14.05	500
30	$13.07	$14.07	80
45	$13.06	$14.09	20
400	$13.05	$14.10	10.4K

MEOW
$13.64 +2.03%

Trade

Who takes your cash when you buy $TSLA?

- Market Makers (Jane Street, Optiver, etc)
- Liquidity is provided in a matter of milliseconds by algorithms
- Highly profitable trading strategies executing thousands of orders on any given day
- Many scales (high frequency: milliseconds, mid-frequency: seconds to minutes, low-frequency: days to weeks to months)
- Different trading strategies have different capacities. Some may only trade millions while others trade billions. Amazon has more $$$ traded in daily volume than soybean futures for instance
- Low-capacity strategies are sometimes ditched by the big players
- Trade currencies, bonds, equities, commodities, weather, rotten tomatoes, anything

Markets are highly self-organizing & efficient

- Information alters market prices
- Highly irregular market patterns are reminiscent of DNA (AGCTAACAG). Analogous to Shannon Entropy
- S&P500 market cap is ~$40-50T
- S&P500 Options Market is ~$50-100T! (Very much like a zero-sum game)
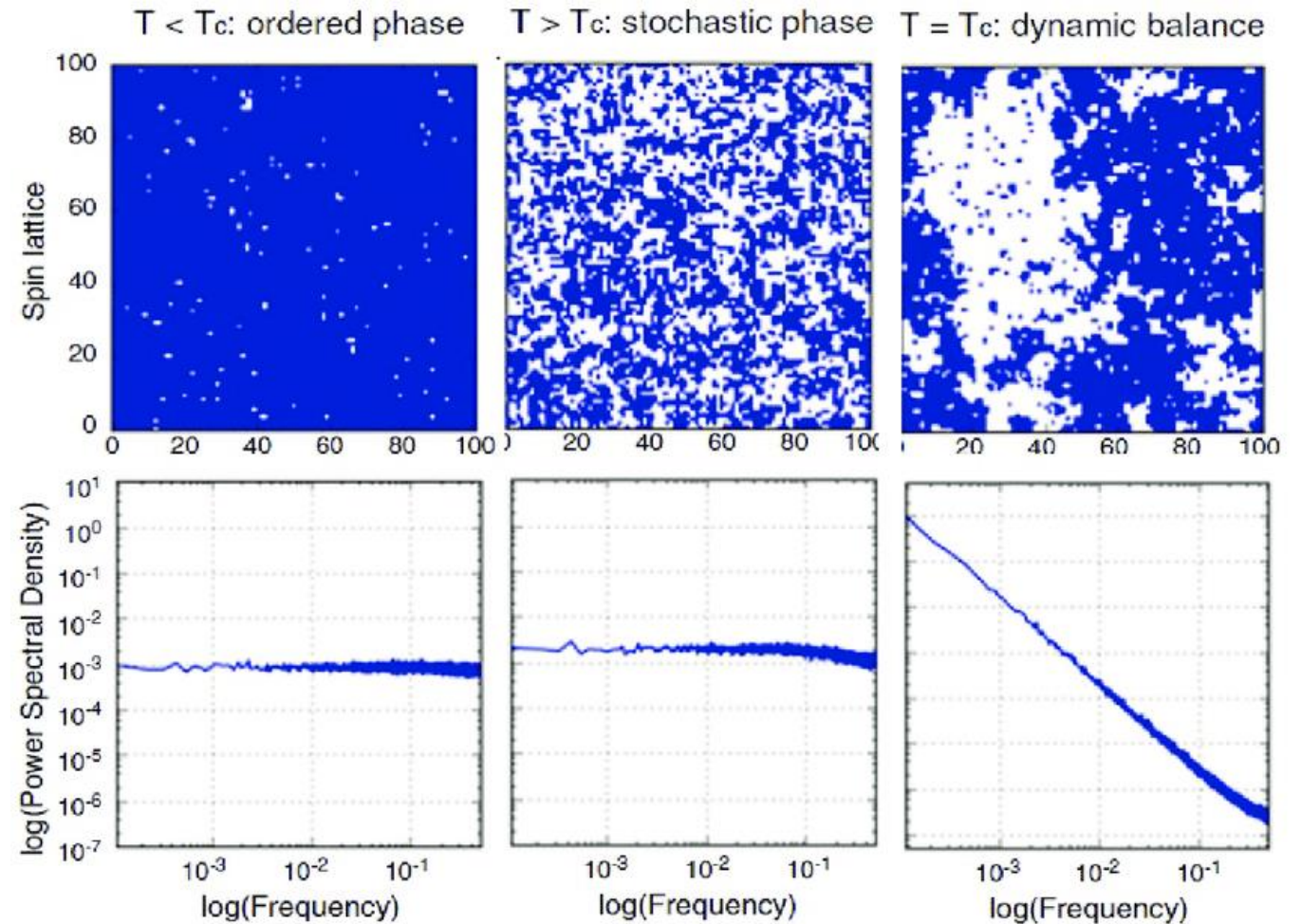- Herding behavior ie imitation

A Physicist's Approach to Modeling Market Bubbles
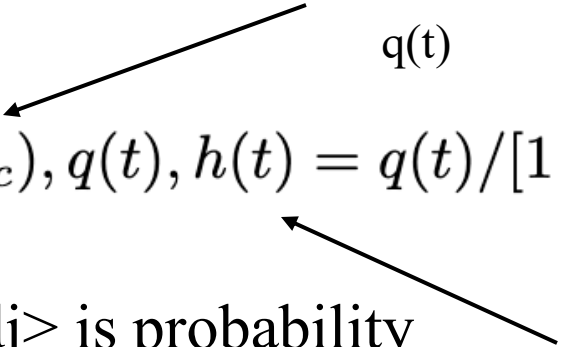
Ising Model! Neighboring spins try to align.

Fix Temperature, and you can calculate the order parameter! Equilibrium

$$H = -\sum_{i \neq j} J s_i s_j$$

Intuition for risk & reward

Random Variable w/ pdf
q(t)

CDF

$$j(t) = \theta(t - t_c), q(t), h(t) = q(t)/[1 - Q(t)], Q(t) = \int_{-\infty}^{t} q(t')dt'$$

<dj> is probability

$$dS = \mu(t)S(t)dt - \kappa S(t)dj \text{ From Geometric BM}$$

Probability of crash happening in next timestep if it has not happened before per unit time

$$\langle dS \rangle = 0 \text{ No Free lunch}$$

$$\implies \mu = \kappa h(t)$$

$$\boxed{S(t) = S(t_0) \exp\left(\kappa \int_{t_0}^{t} h(t')dt'\right)}$$

Toy model solution has a BOOM before the bust. Booming investments can have big hidden tail risk

Sornette's Model

NOTE: "All models are wrong but some are useful"

Inspired by models for earthquake prediction & phase transitions. Derived from modeling market participants as forming a self-similar network with herding.
- Sornette was a professor at UCLA when he came up with this. Bet and profited from dot-com bubble!

$$\ln S(t) = A + B(t - t_c)^\alpha + C(t - t_c)^\alpha \cos\left[\omega \ln(t - t_c) - \phi_1\right]$$
$$+ D(t - t_c)^\alpha \cos\left[2\omega \ln(t - t_c) - \phi_2\right]$$

Analogous to temp in stat mech.

Didier Sornette. *Why Stock Markets Crash: Critical Events in Complex Financial Systems*. Princeton University Press, 2017.

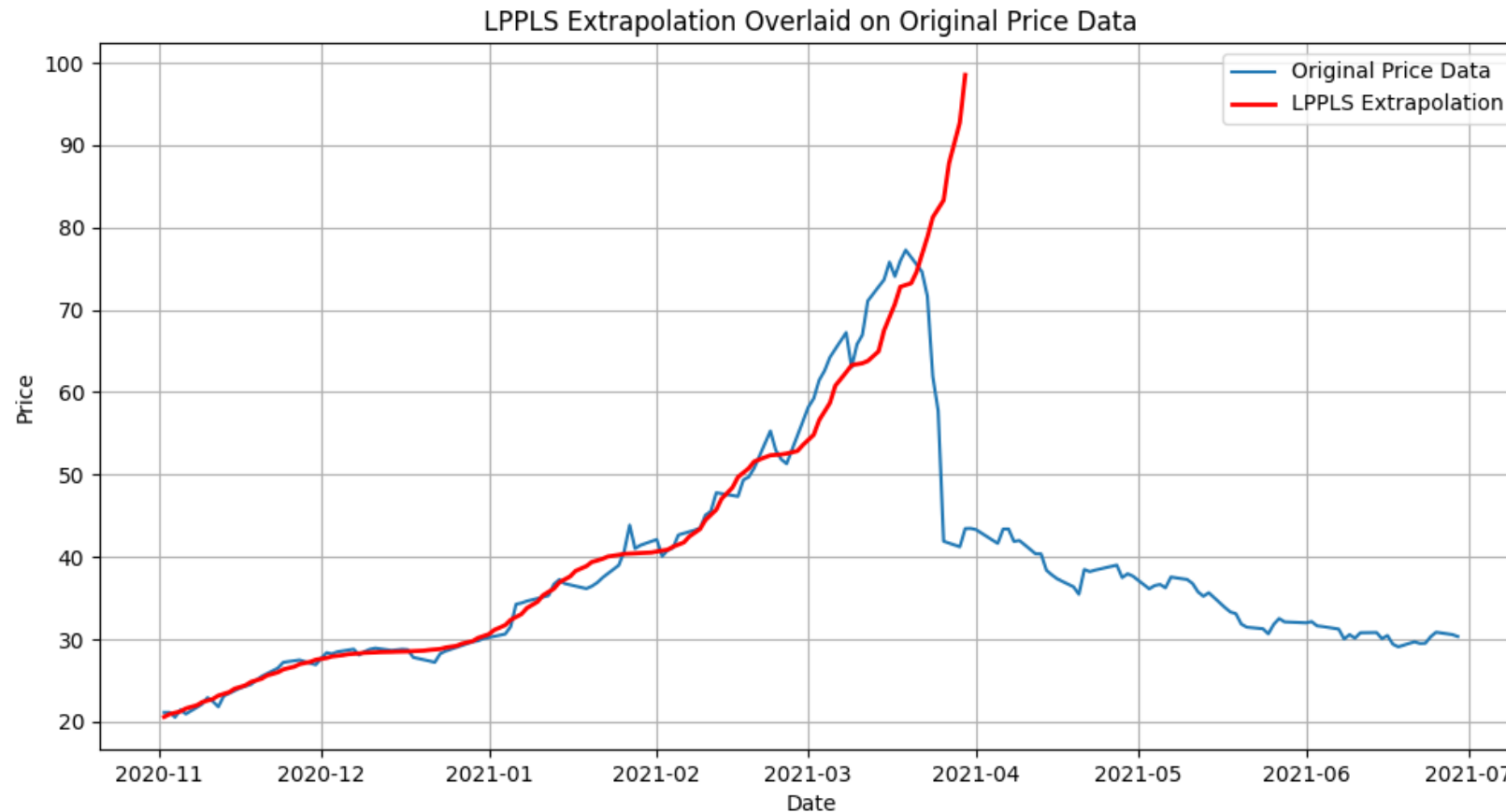Johannes Voit. *The statistical mechanics of financial markets*. Springer, 2003.

Examples   Calculations by Sornette

Prediction of Crashes
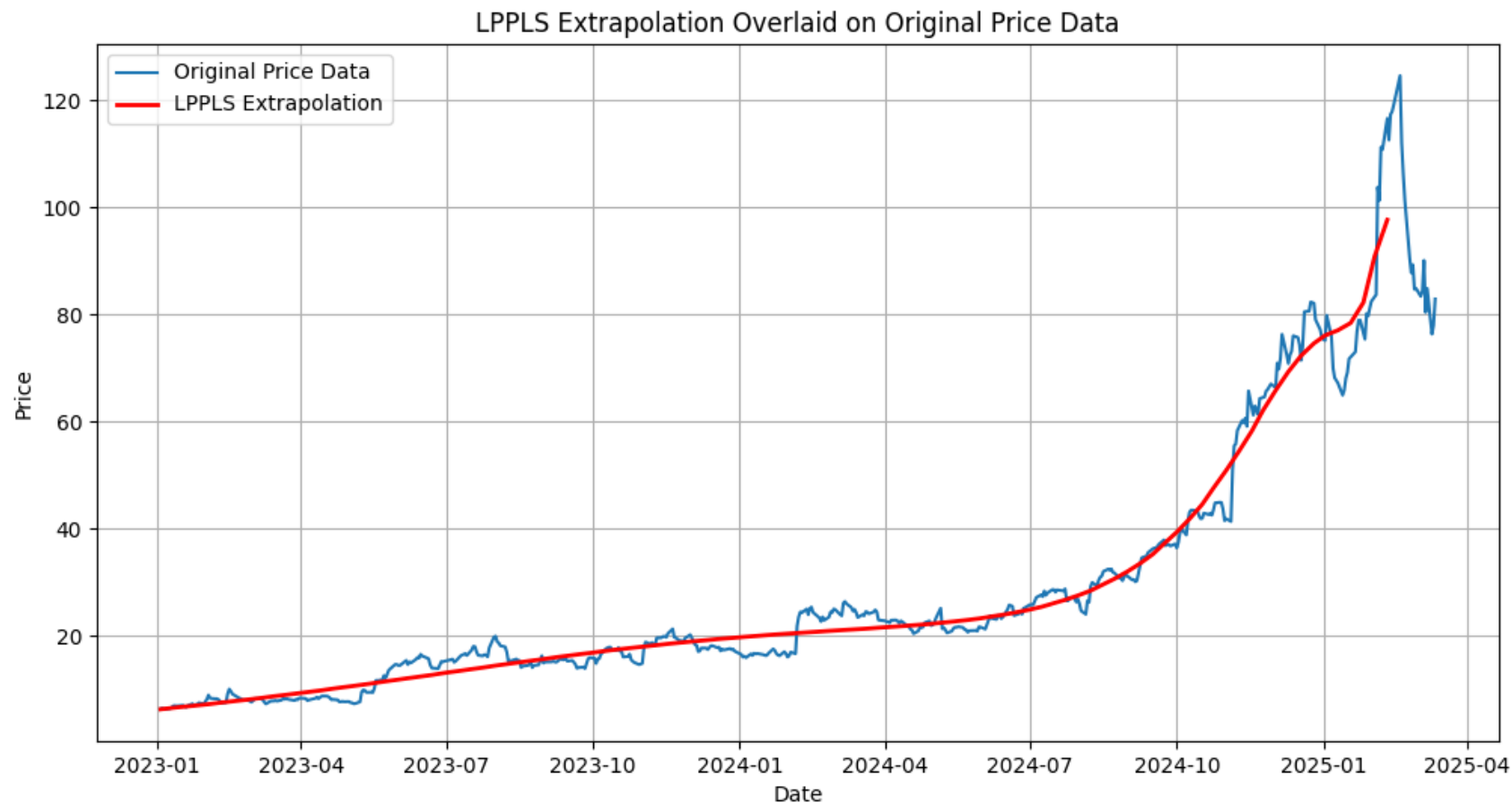
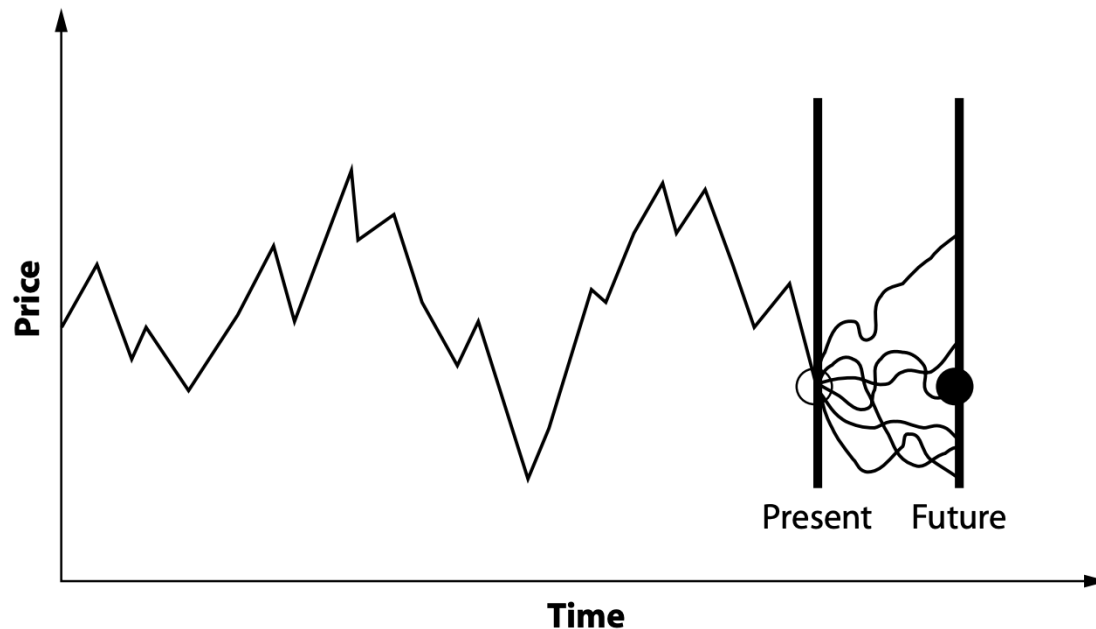$WBD in March 2021. Formerly Viacom. Same thing with Discovery stock at the time.

LPPLS Extrapolation Overlaid on Original Price Data

More Recently

$PLTR. Fitting stopped at Feb 10. Crash occurred Feb 18

LPPLS Extrapolation Overlaid on Original Price Data

How to profit from forecasts: What is an option?

- "GF doesn't care about poker theory. She cares if you made money"

- The price of a contract is the expected value of profit!

- Black-Scholes assumes Gaussian returns

Options Pricing

- Example: Apple stock is at $100. Call Contract for strike price $110. Let's you buy at $110 and sell at the market price. What is the fair price ie expected value? Under what probability measure?

- Standard formula is Black-Scholes

- Options market is multiples of the stock market

$$dS = \mu S_t dt + \sigma S_t dW_t$$

Terminal Boundary Condition!

$$V(S, T) = \max(S - K, 0)$$

$$\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + rS\frac{\partial V}{\partial S} - rV = 0$$
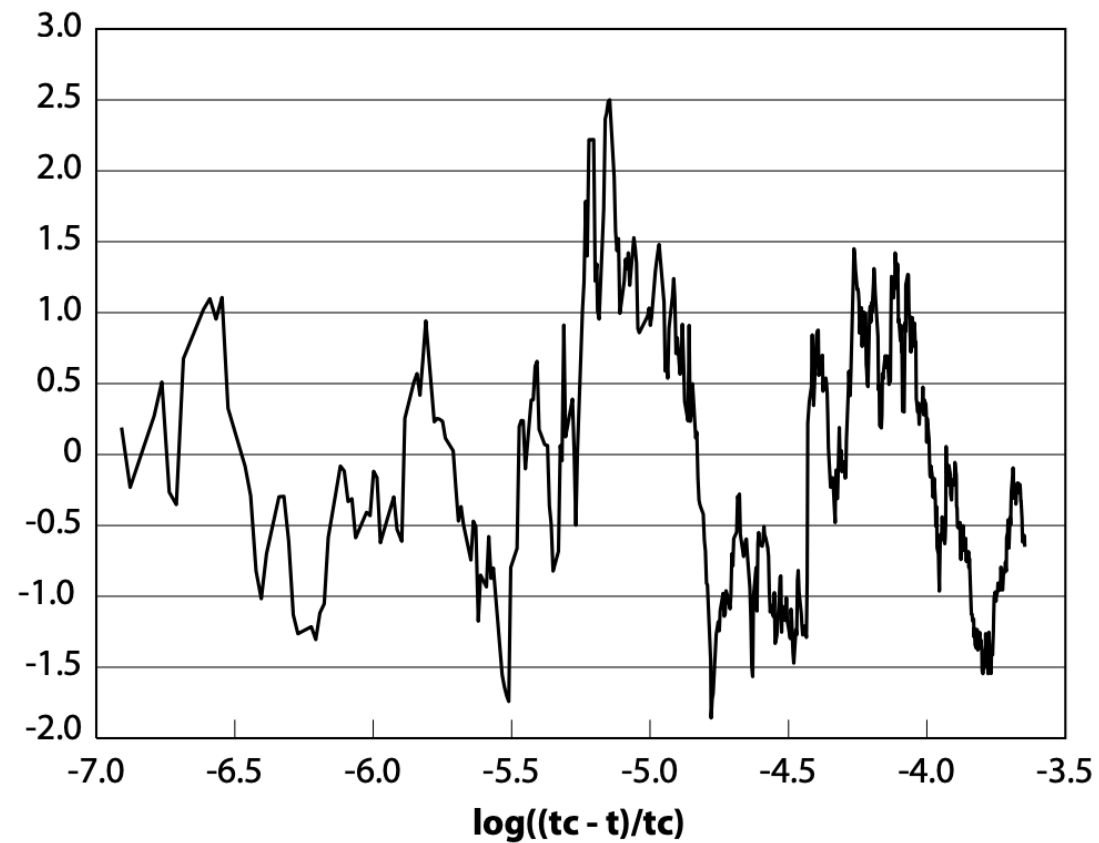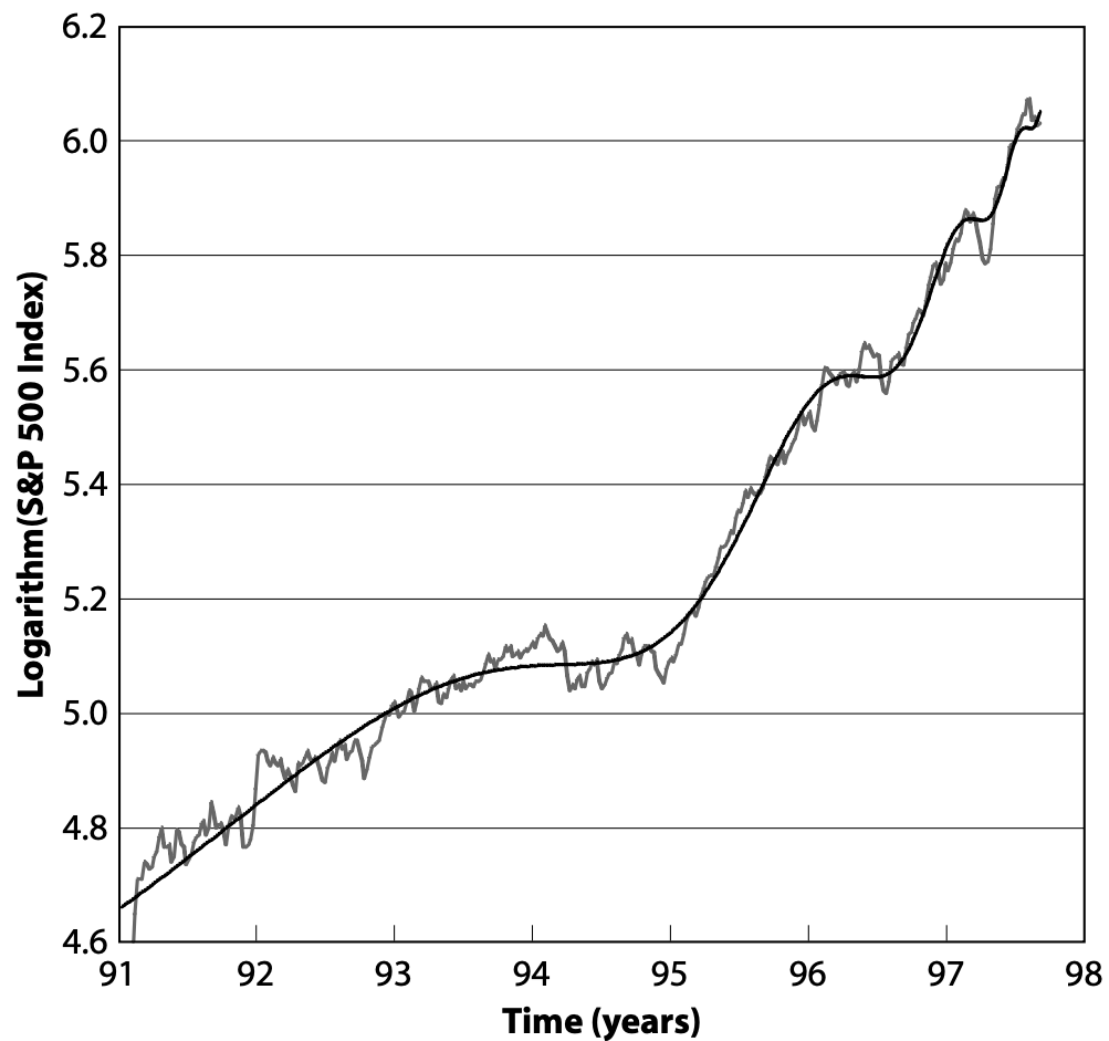
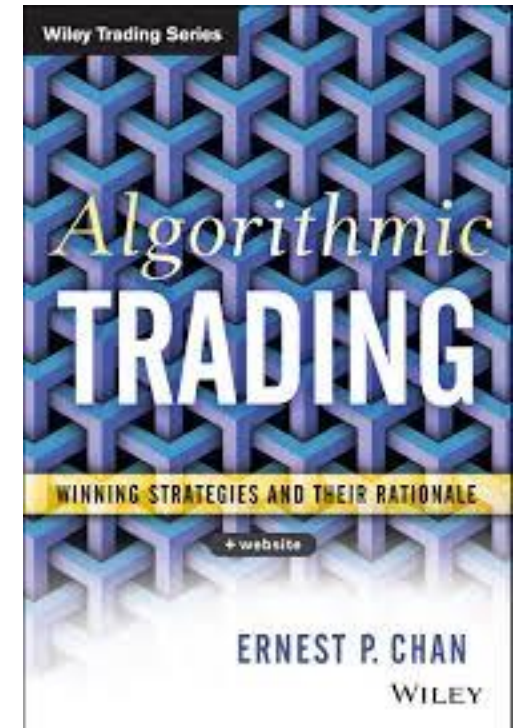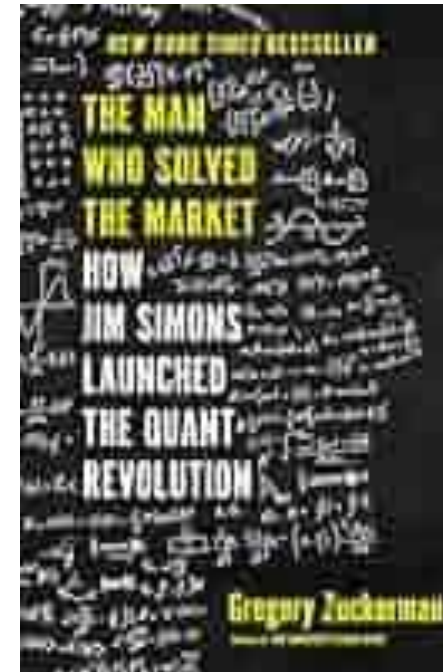Many more types of models, especially proprietary nonpublic. What if the walk is formed from a Cauchy distribution? Path Integral techniques have been applied
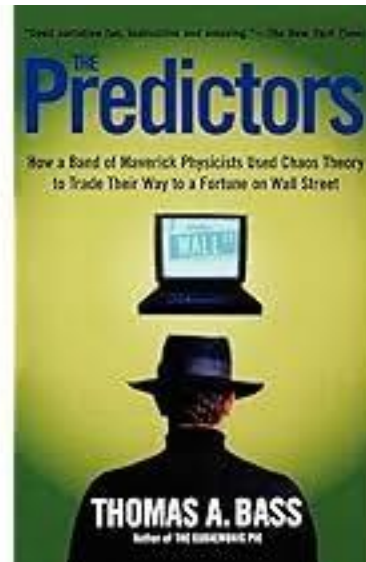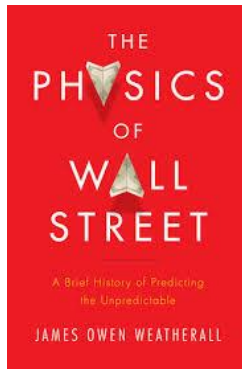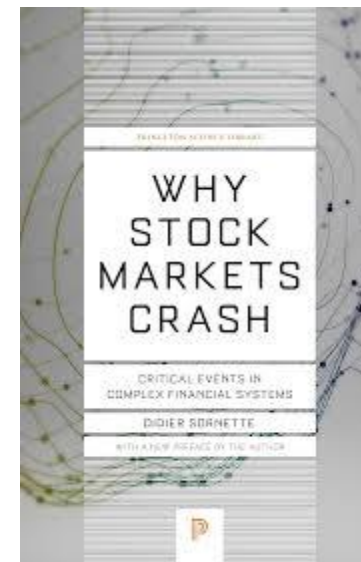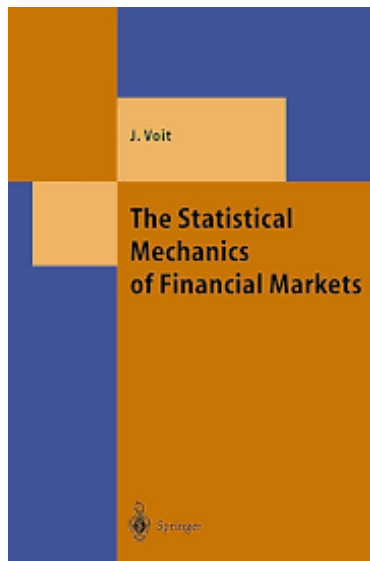
Option Price Trajectories
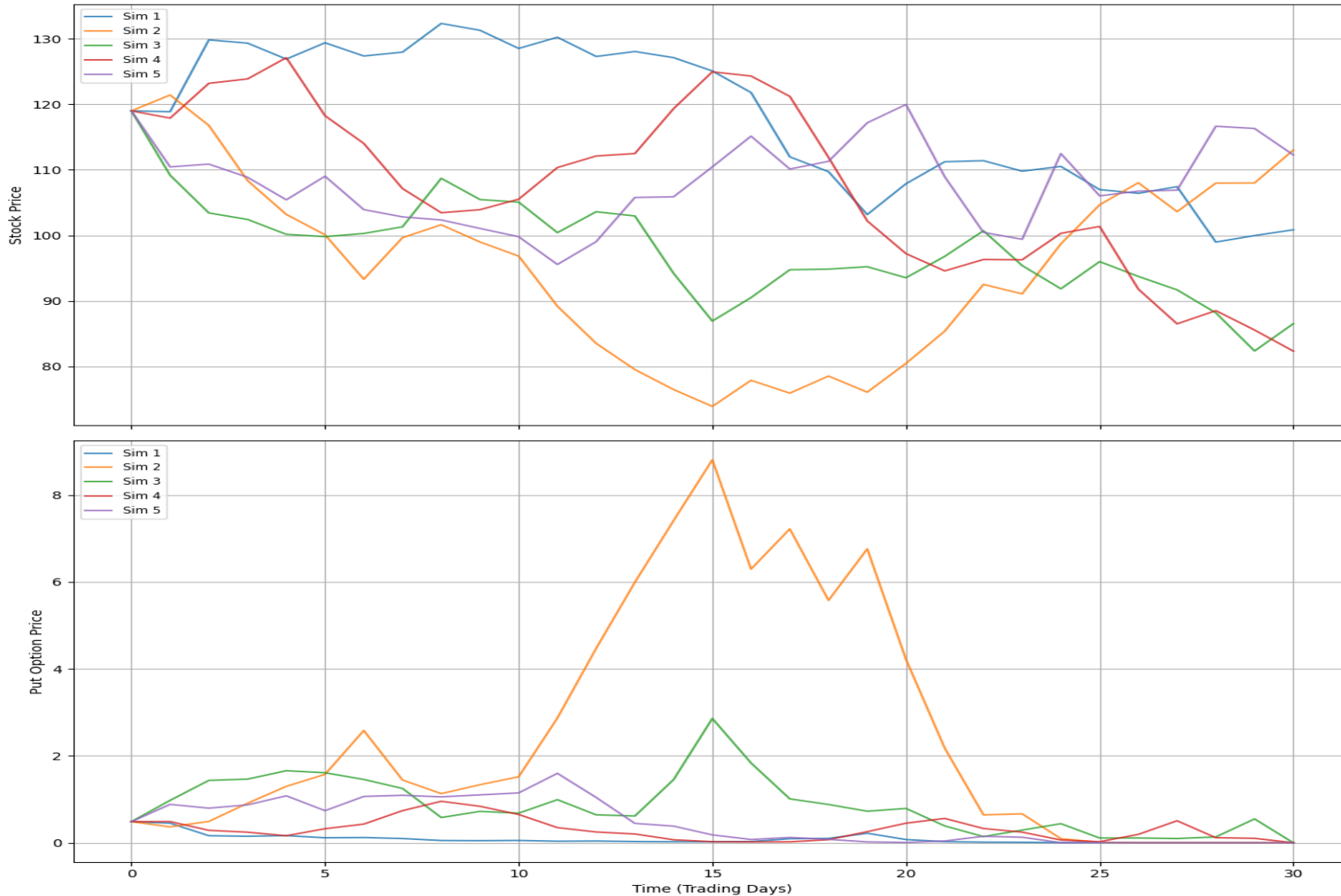
Reward is sometimes multiples of what you are risking.

Put option expiring in a month. Strike of $80

Further Reading

Physics PhD turned hedge fund manager

Sornette's Book. Great Read

Thanks for listening

CEO of Goldman Sachs