Motivating a theory of Superconductivity:

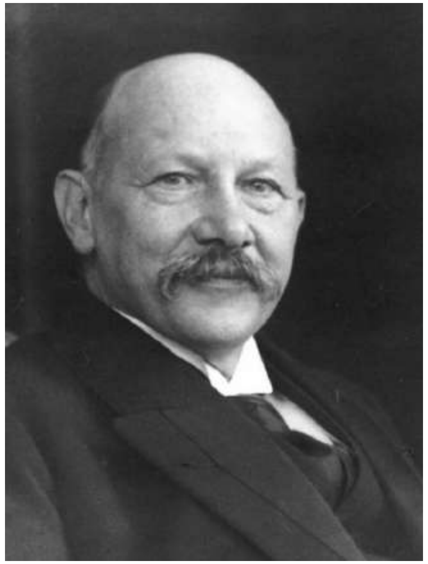
(originally titled “Understanding Superconductivity”)

Sparsh Mishra

Physics Concerto 29/01/2025

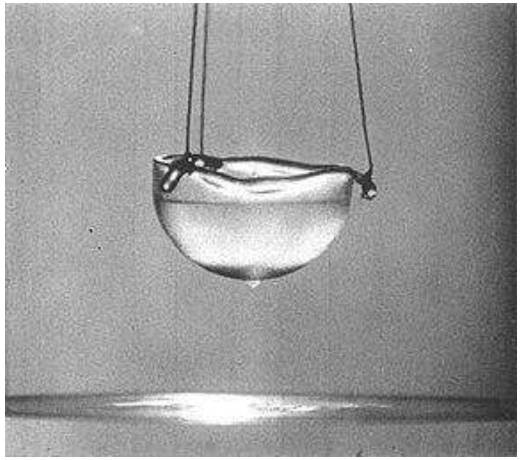
The beginning

Heike Kamerling Onnes at Leiden, Netherlands



First Liquified Helium (1.4K) in 1908

(previous lowest temperature: liquid H_2 - 14 K)



“Father of Cryogenics”

Photo drawn by his nephew- He was a portrait painter- Not bad right?

Running experiments to see Does resistance diverge at 0 K or go to some constant.

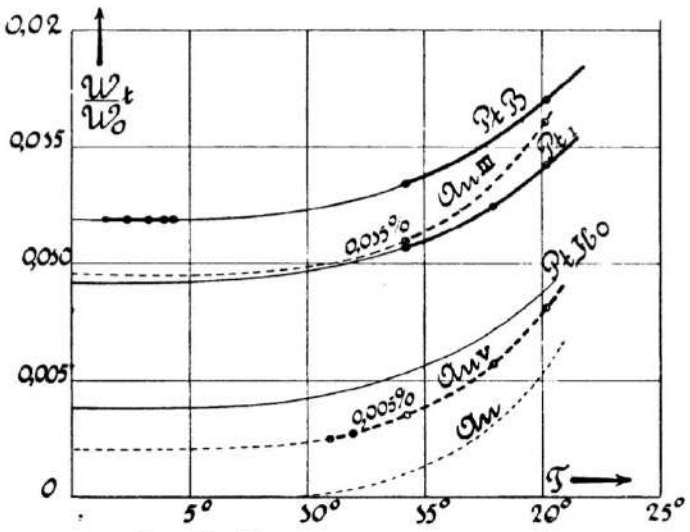


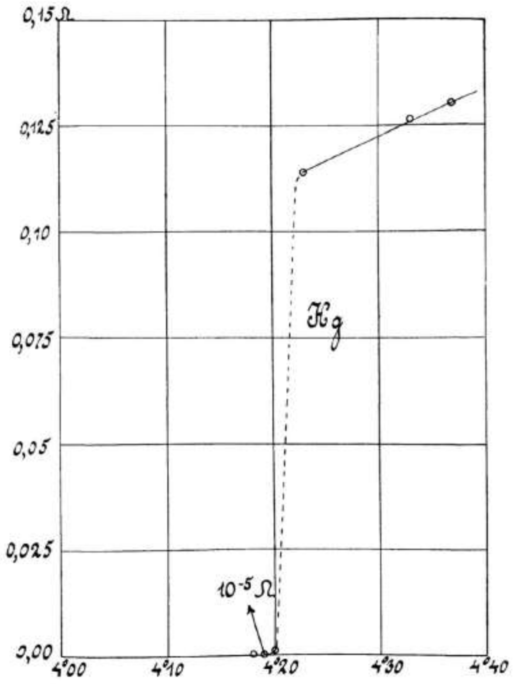
Fig. 3. Resistances of platinum.

Discovered SC while running the experiment for Hg

Got the Nobel prize in 1913

Current in a superconductor doesn't decay

In 1954 BS Chandrasekhar and DM James ran current through a SC ring. Could not detect decay even after <u>2.5 years</u>!



Through Measurement to Knowledge: The Selected Papers of Heike Kamerlingh Onnes

Theories of Superconductivity*

Unsuccessful attempts to understand superconductivity by:

Depressing/Inspiring Quote by einstein on a paper on this

...nature is a merciless and harsh judge of the theorist's work. In judging a theory, it never rules „Yes" in best case says „Maybe", but mostly „No". In the end, every theory will see a „No".

Einstein, Albert. "Theoretical remark on the superconductivity of metals." *arXiv preprint physics/0510251* (2005).

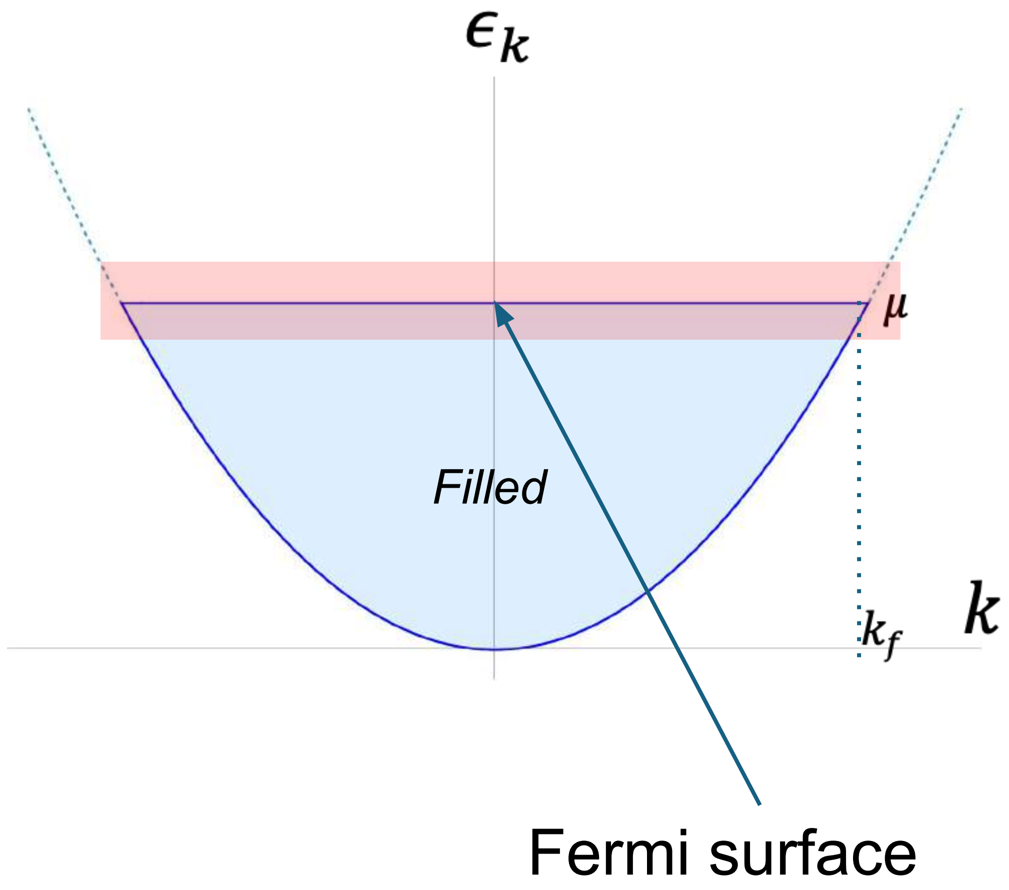
*Thanks, Bishoy, for the slide

Goal for today

1) See why superconductor is a weird state of matter

2) Motivate a theory of superconductivity

A bit about Metals



Start with some energy levels (continuum)

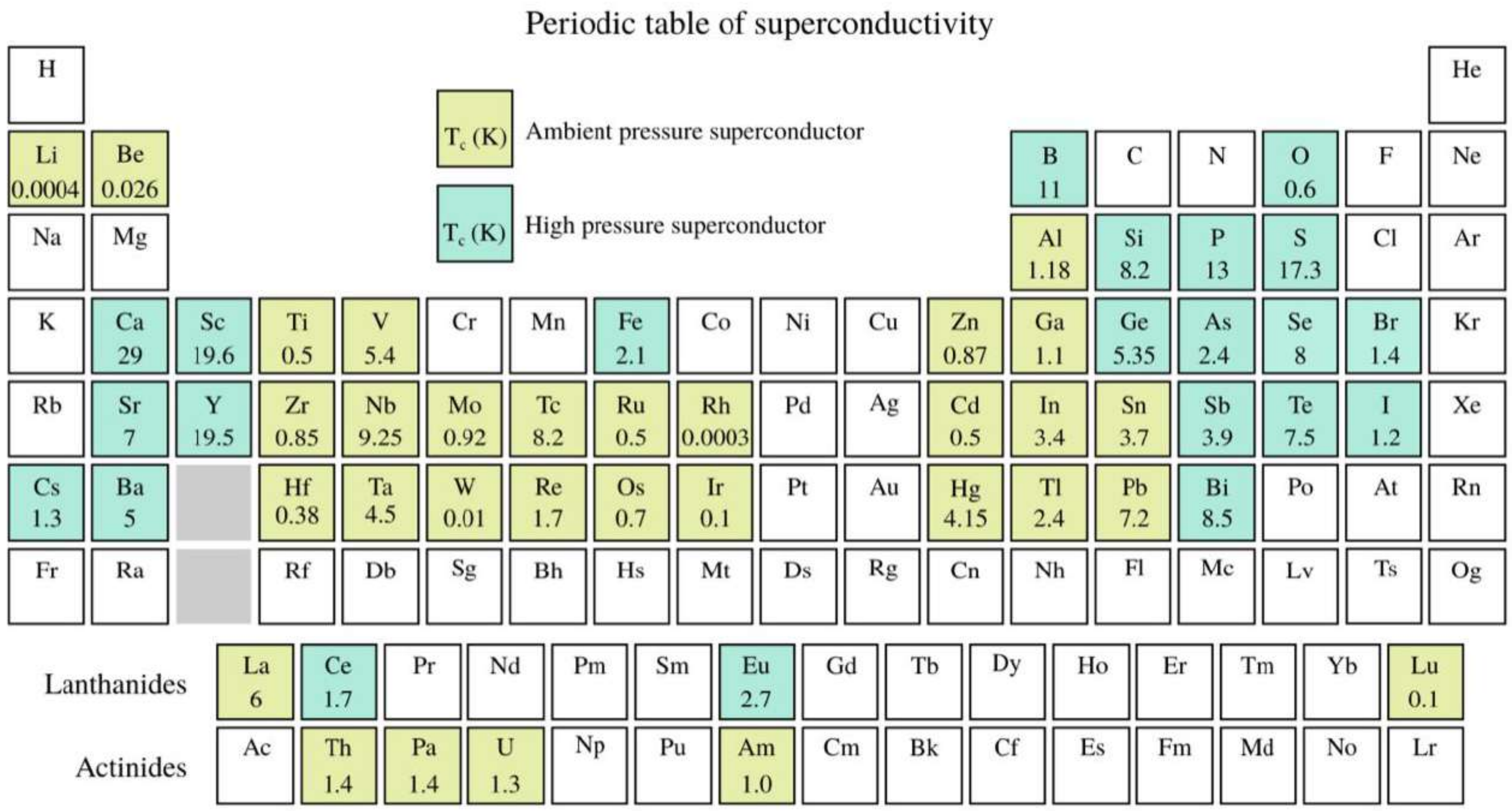
Each electron occupies an energy level (according to the Pauli principle)

Highest energy occupied is called the fermi-energy or the chemical potential. Fermi surface

The fermi-energy is typically very large $\sim 10^4\ K$

Transport/ scattering/...etc. All happens near the fermi surface

How many superconductors?

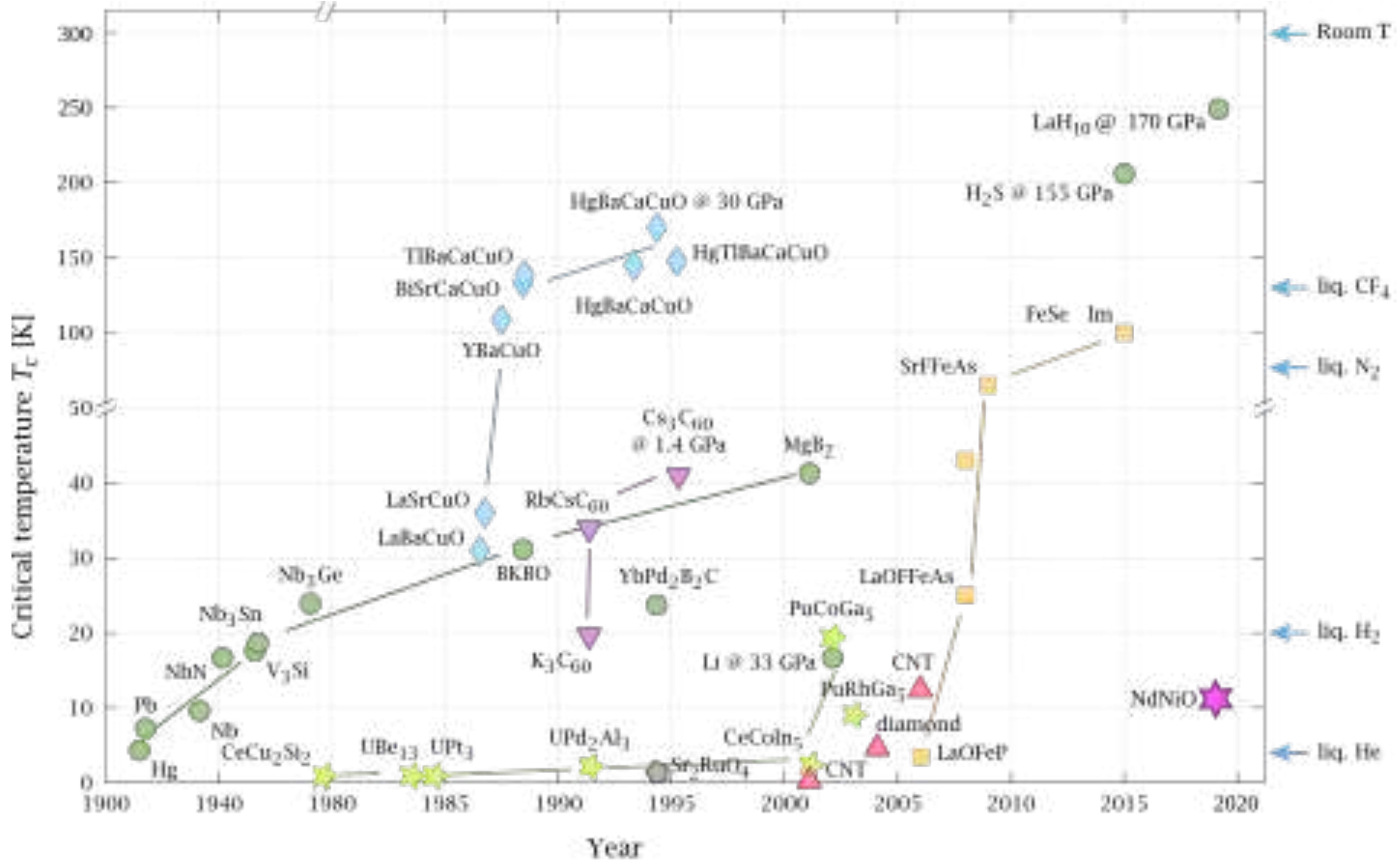


Under ambient pressure 33 elemental SC

All metals before SC

(Free electron models work well)

A lot of others but I’m not going to talk about them!

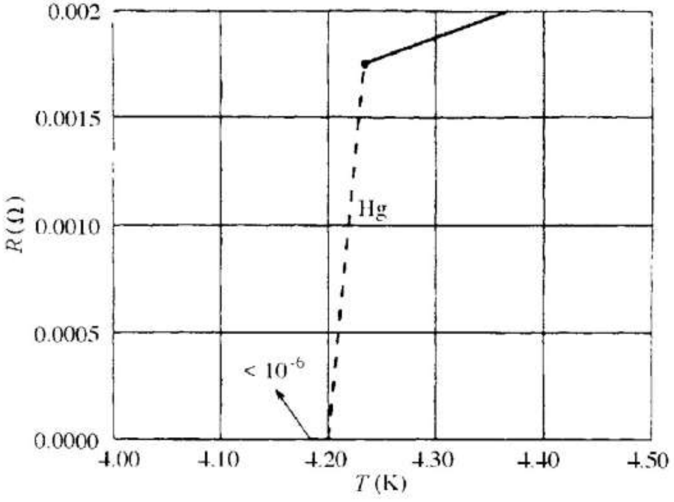


Timeline of superconducting materials. Colors represent different classes of materials:

- BCS (dark green circle)
- Heavy fermion-based (light green star)
- Cuprate (blue diamond)
- Buckminsterfullerene-based (purple inverted triangle)
- Carbon-allotrope (red triangle)
- Iron-pnictogen-based (orange square)
- Strontium ruthenate (grey pentagon)
- Nickel-based (pink six-point star)

Experiments

1) Vanishing DC resistivity



Perfect metal?

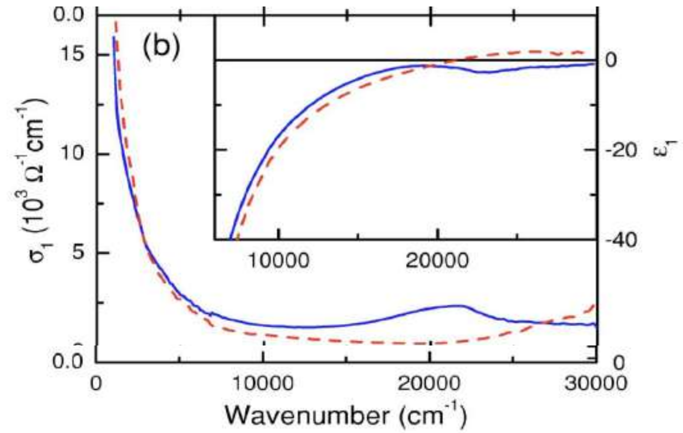
Metals typically have Re AC Conductivity peak at 0 because of free carriers

Classical Free electrons ~

$$m\frac{d^2r}{dt^2}+\frac{m}{\tau}\frac{dr}{dt}=-eE(t)$$

$$\Rightarrow \sigma = e\frac{dr}{dt}\propto\frac{1}{1+\tau^2\omega^2}$$

Eg. MgB_2



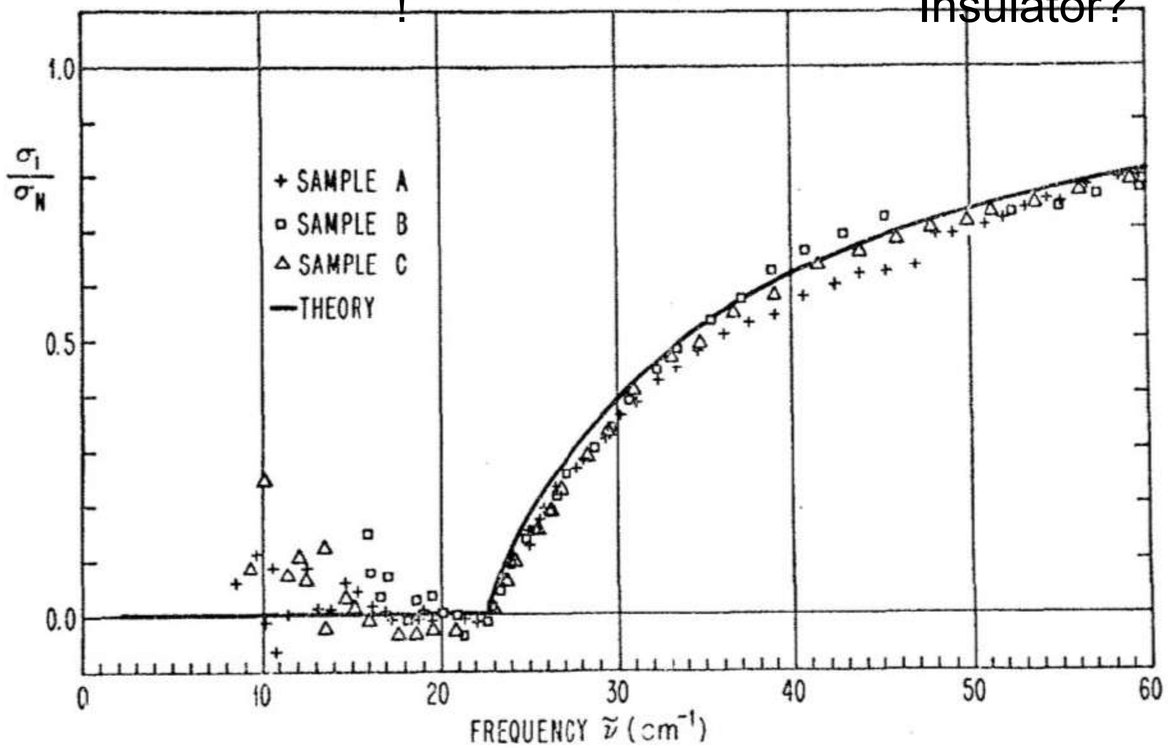
Guritanu, V. *et al. Phys. Rev. B* **73**, 104509 (2006).

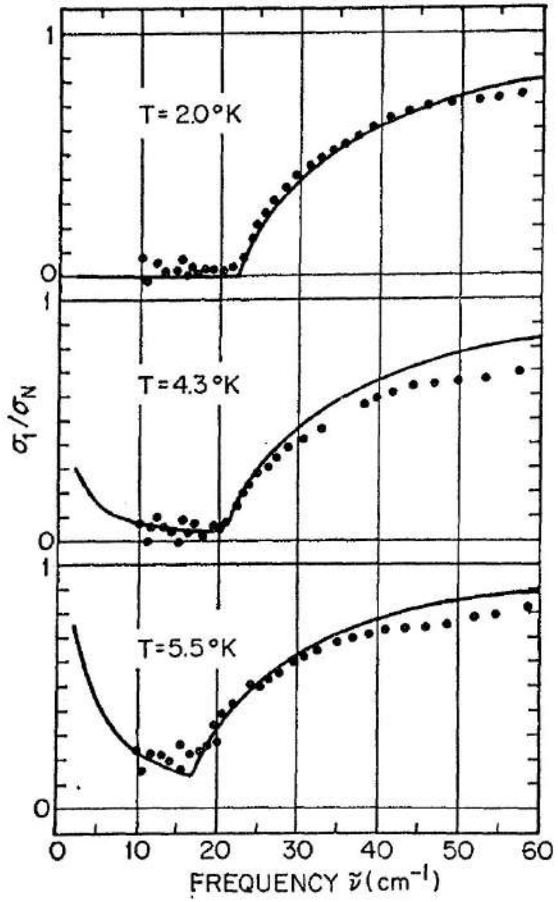
Re AC conductivity:

Temperature dependence

Seems to be a gapped !

No free carriers? Insulator?





$1\ cm^{-1} \sim 0.1\ meV$

Gap of $1 - 2\ meV$

Visible light: $1.6 - 3.2 eV$

X ray energy $100\ eV\ - 100\ keV$

Palmer, L. H. & Tinkham, M. *Phys. Rev.* **165**, 588–595 (1968).

Specific heat T- dependence

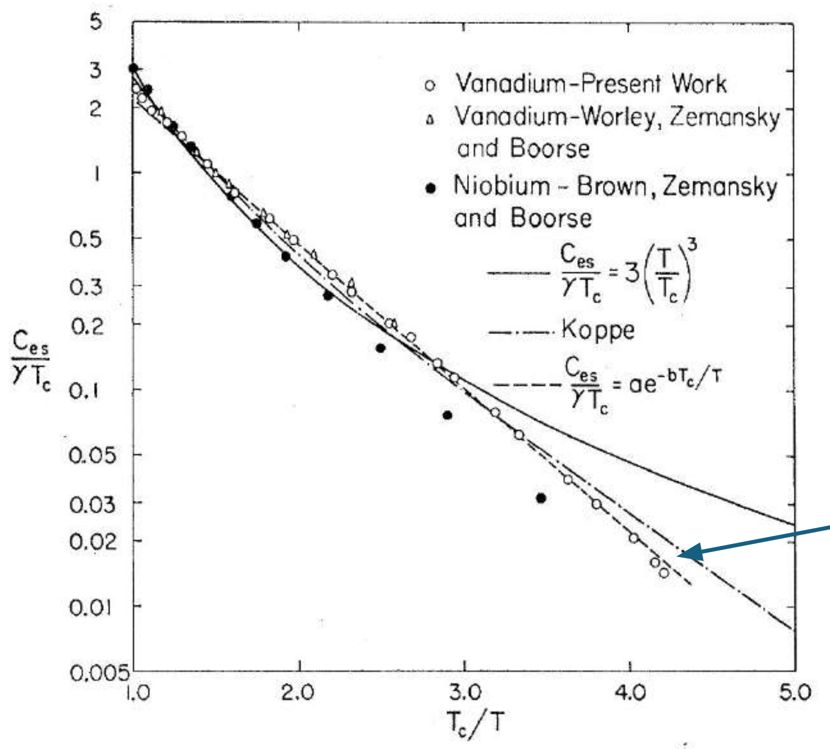


FIG. 1. Temperature dependence of the electronic specific heat of a superconductor.

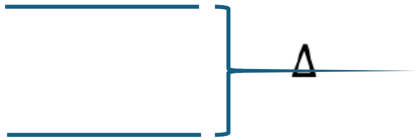
Electronic Specific heat of Va vs 1/T

For most metals $C_v \propto \alpha_{elec}T + \beta_{phon}T^3$

*Phonons removed

$$C_v \propto -\frac{1}{T} \Rightarrow C \propto e^{-b/T}$$

For a two-level system



$$Z = 1 + e^{\beta\Delta}$$

$$\Rightarrow C_v = \partial_T \langle E \rangle = \partial_T \left(\frac{\Delta e^{\beta\Delta}}{1 + e^{\beta\Delta}} \right) \rightarrow \propto \frac{e^{-\frac{\Delta}{T}}}{T^2}$$

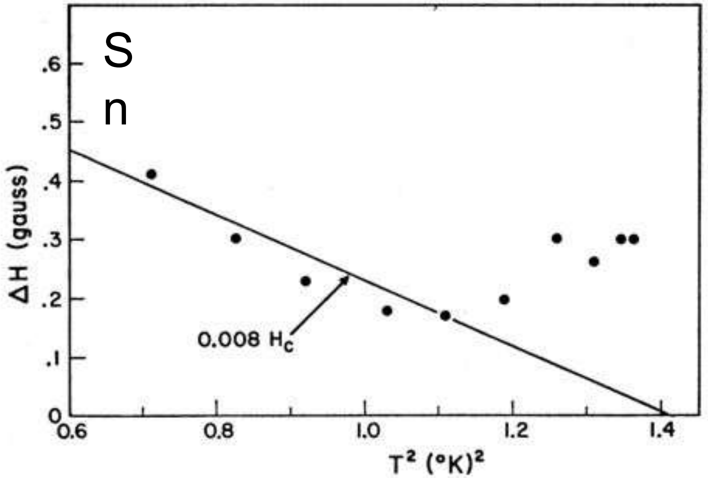
Low T limit

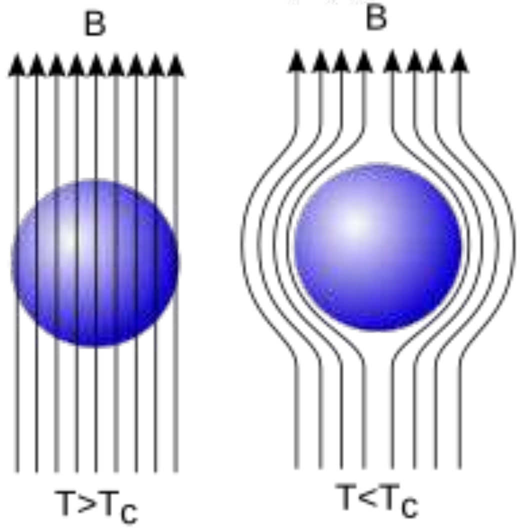
At the superconducting transition, the system generates a gap to become stable

Corak, W. S., Goodman, B. B., Satterthwaite, C. B. & Wexler, A. *Phys. Rev.* **96**, 1442–1444 (1

Critical magnetic field and Meissner effect

B-fields kill superconductivity





Cochran, J. F. & Mapother, D. E. *Phys. Rev.* **111**, 132–142 (1958).

Perfect metal:

$j = \sigma E$

If $\sigma = \infty \Rightarrow E = 0$

$\partial_t B = \nabla \times E = 0 \Rightarrow B = cte$

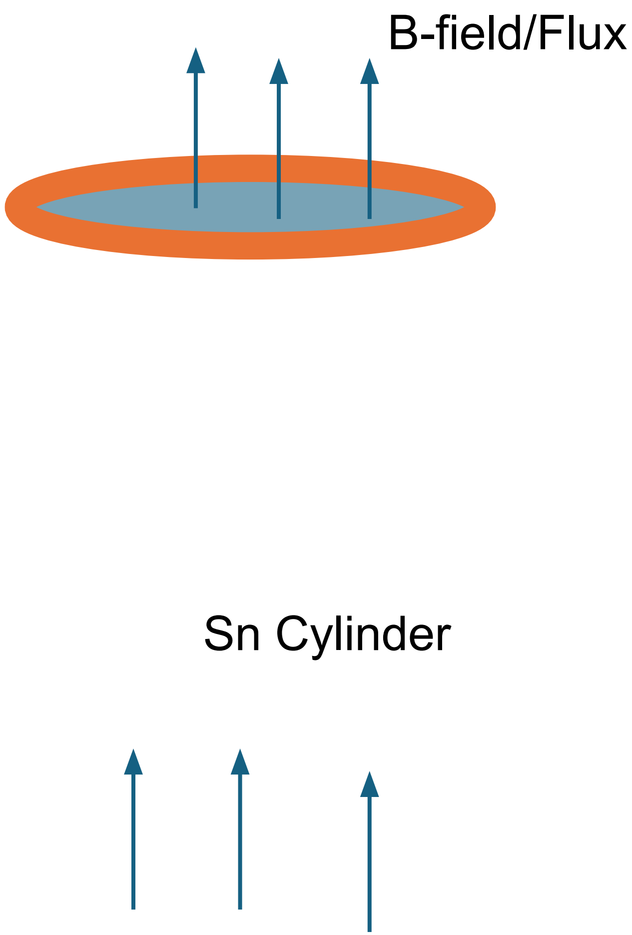
"magnetic field freezes in"

A SC pushes out magnetic field to an extent after which is gives up and becomes a metal again

Tells us that SC don't like breaking Time reversal symmetry!

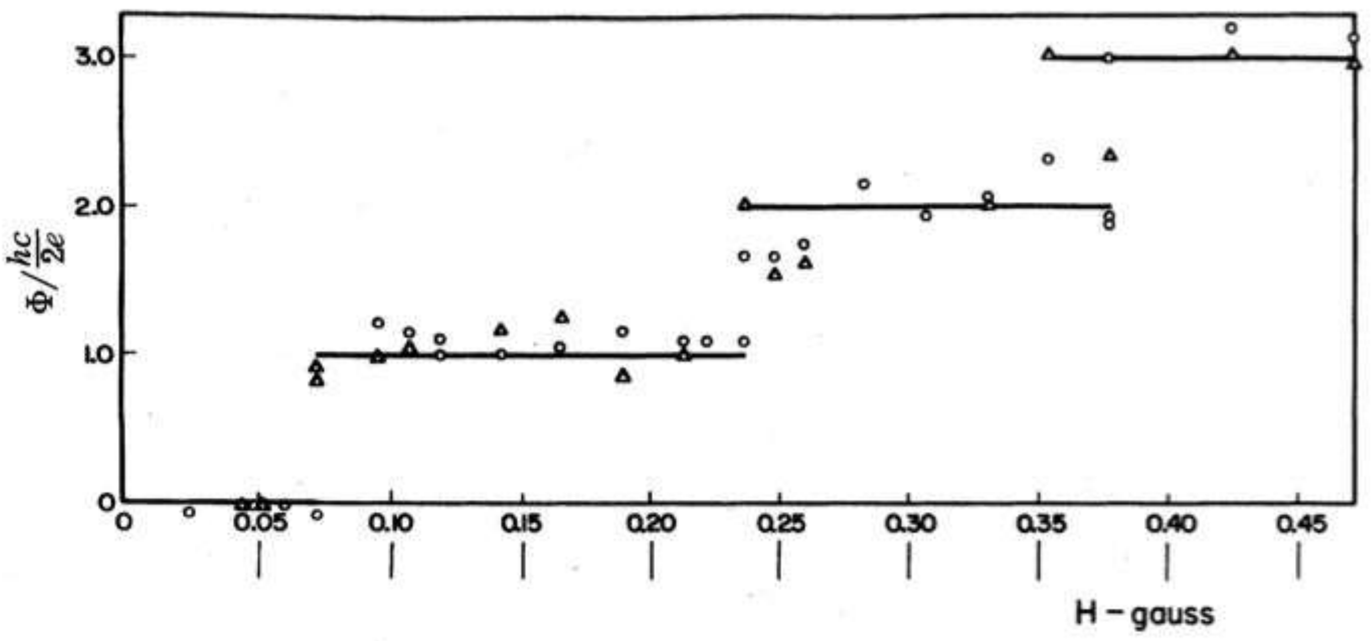
$(Maxwells\ equations:\ t \rightarrow -t, B \rightarrow -B)$

Quantized Flux quantum



1) Take hollow Tin Cylinder.(Before Cooling)
2) Apply magnetic field in one direction and Cool it
3) Flux passes through hollow area.
4) Turn off B field.
5) Measure Flux (usi

Deaver, B. S. & Fairbank, W. M. *Phys. Rev. Lett.* **7**, 43–46 (19



$\frac{hc}{2e}$ Why is there this factor of 2?

For an electron on a ring

$$\psi(x_i) \to e^{2\pi i \frac{e}{hc} A(x_i)} \psi(x_i)$$

To be single valued:

$$\int dl \cdot A = \frac{\Phi}{\frac{hc}{1e}} = n \in Z$$

$$m\frac{dv_s}{dt} = -eE$$

Persistent currents and magnetic flux

$$J_s = -n_s e\, v_s$$

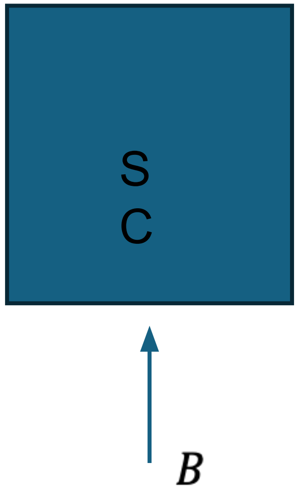
$$\frac{d}{dt}J_s = -n_s e\frac{dv_s}{dt} = \frac{n_s e^2}{m}E \quad \xrightarrow{\nabla\times E = -\frac{1}{c}\frac{\partial B}{\partial t}} \quad \frac{d}{dt}\left(\nabla\times J_s + \frac{n_s e^2}{mc}B\right) = 0$$

If $\left(\nabla\times J_s + \frac{n_s e^2}{mc}B\right) = 0$ $\xrightarrow{\nabla\times B = \frac{4\pi}{c}j_s}$ $\nabla^2 B = \frac{1}{\lambda_L^2}B, \quad \lambda_L = \sqrt{\frac{4\pi n_s e^2}{mc^2}}$

(London equation
)

Solutions such as $B_x(z) = B_{0x}e^{-z\lambda}$

(Meissner
effect)



But what does this
mean?

$$\nabla\times J_s + \frac{n_s e^2}{mc}B \Rightarrow J_s = \frac{n_s e^2}{mc}A + \nabla\phi \quad \xrightarrow[\text{Coulomb gauge } \nabla\cdot A = 0]{\nabla\cdot J_s = 0} \quad J_s = \frac{n_s e^2}{mc}A$$

Facts so far

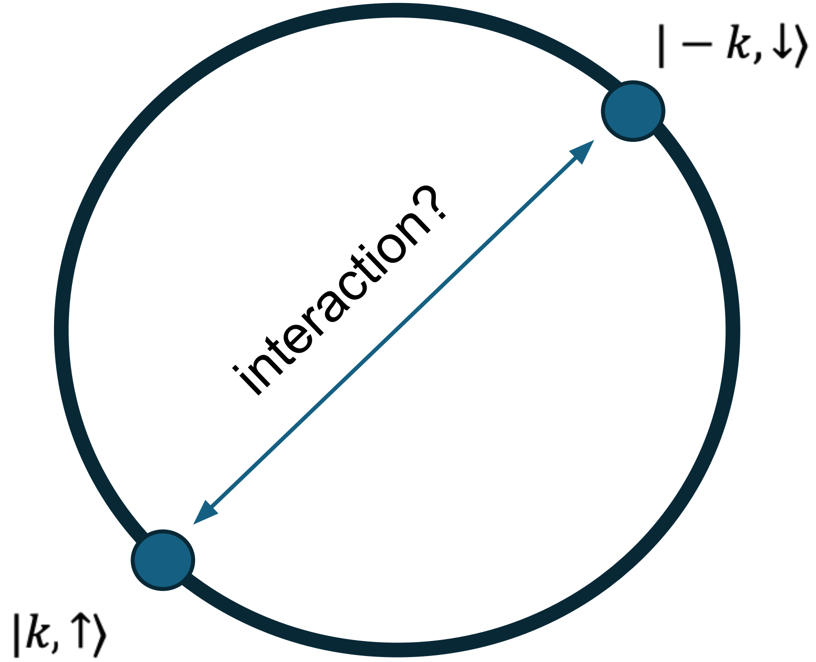
All metallic before transition

Flux quantized in units of 2 electron charge ~ bound pair

Breaking Time reversal symmetry destroys the superconductor

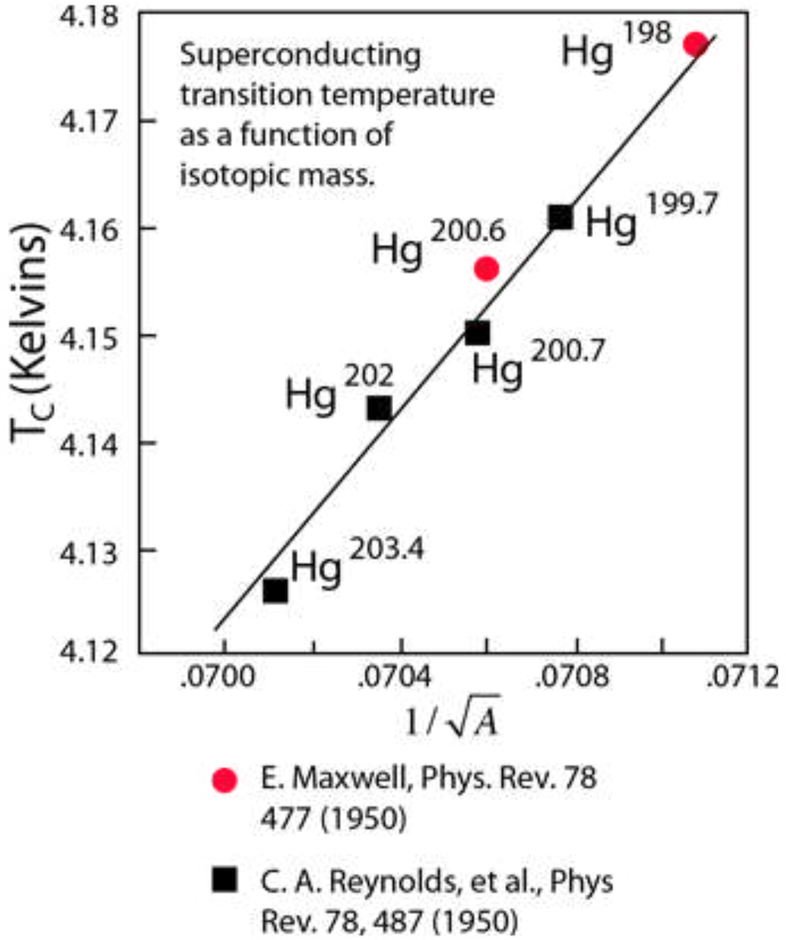
Gap in spectrum

What kind of interaction?
Attractive

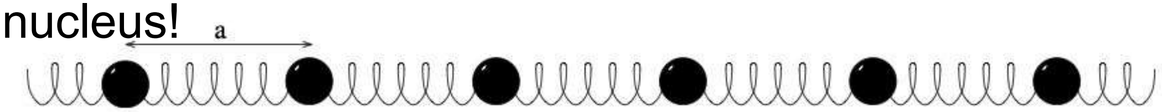


Fermi Surface

Attractive interaction- Isotope effect



Electrons not really affected by the mass of nucleus!



$$H = \sum_n \frac{p_n^2}{2m} + \frac{\lambda}{2}\sum_n (u_n - u_{n-1})^2$$

Solve using Newtons laws/Lagrangian →

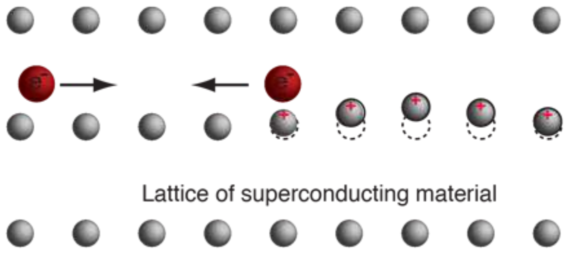
(plane wave solutions)

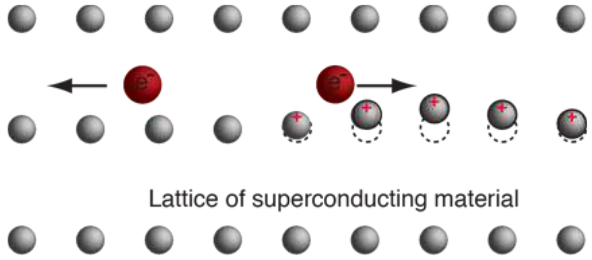
$$u_n = A\, e^{-i\omega t - ikna}$$

$$\omega = 2\sqrt{\frac{\lambda}{m}}\left|\sin\left(\frac{ka}{2}\right)\right|$$

SC has Something to do with Lattice vibrations a.k.a Phonons

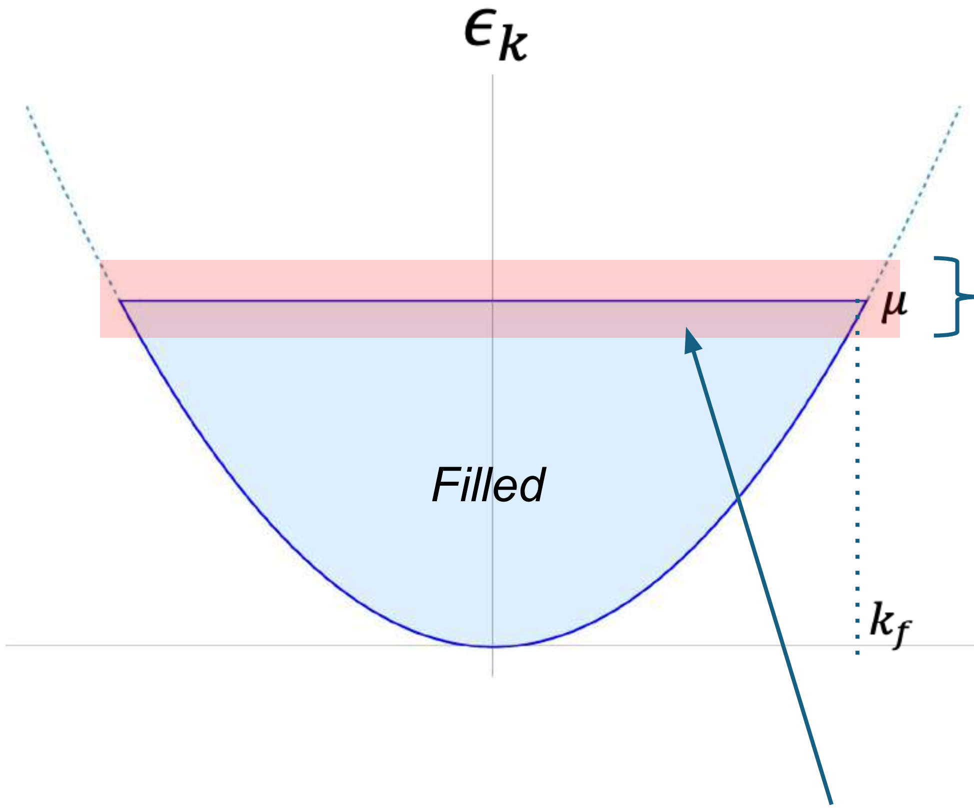
Phonon exchange = attractive interactions





http://hyperphysics.phy-astr.gsu.edu/hbase/Solids/coop.html

Schematic energy of a free electron (actually it's a 3D parabola)



Max phonon energy $\equiv \omega_D \ll \mu$

$\Rightarrow$ Attractive attraction only for a small energy window around fermi energy

k

Fermi-wavevector

Fermi surface (where all excitations happen)

Variational ground state: $\langle N \rangle$ electron pairs

Explaining the spectral gap

$$|\Psi_{BCS}\rangle = \prod_{k \in FS} (u_k + v_k \, c^\dagger_{k\uparrow} c^\dagger_{-k\downarrow}) |FS\rangle,. \quad u_k^2 + v_k^2 = 1$$

superposition of $\langle N \rangle$ electron pairs with COM momentum 0

$$\bar{N} = \langle N \rangle = \langle \Psi_{BCS} | N | \Psi_{BCS} \rangle$$

u_k, v_k: variational parameter that we can tune

Try to see what the spectrum of the system is

$$H = T + U = \sum_{ks} \epsilon_k c^\dagger_{sk} c_{sk} + \sum_{kk'q} V_{kk'q} c^\dagger_{k'+qs} c^\dagger_{k-qs'} c_{ks'} c_{k's},$$

$$V_{kk'} = \begin{cases} -g < 0 & |\epsilon_k - \mu| < \omega_D \\ 0 & otherwise \end{cases}$$

$$\rightarrow \sum_{ks} \epsilon_k c^\dagger_{sk} c_{sk} - g \sum_{kk'} c^\dagger_{k'\uparrow} c^\dagger_{-k'\downarrow} c_{k\uparrow} c_{-k\downarrow}$$

$$E = \langle \Psi_{BCS} | H | \Psi_{BCS} \rangle - \mu \bar{N} = \sum_k 2(\epsilon_k - \mu) v_k^2 - g \sum_{kk'q} v_k u_k v_{k'} u_{k'}$$

Minimizing wrt . u_k, v_k $\delta E = \delta(\langle\Psi_{BCS}|H|\Psi_{BCS}\rangle - \mu\bar{N}) = 0$

With the constraints $u_k^2 + v_k^2 = 1, \langle N\rangle = \bar{N}$

After a bunch of algebra →

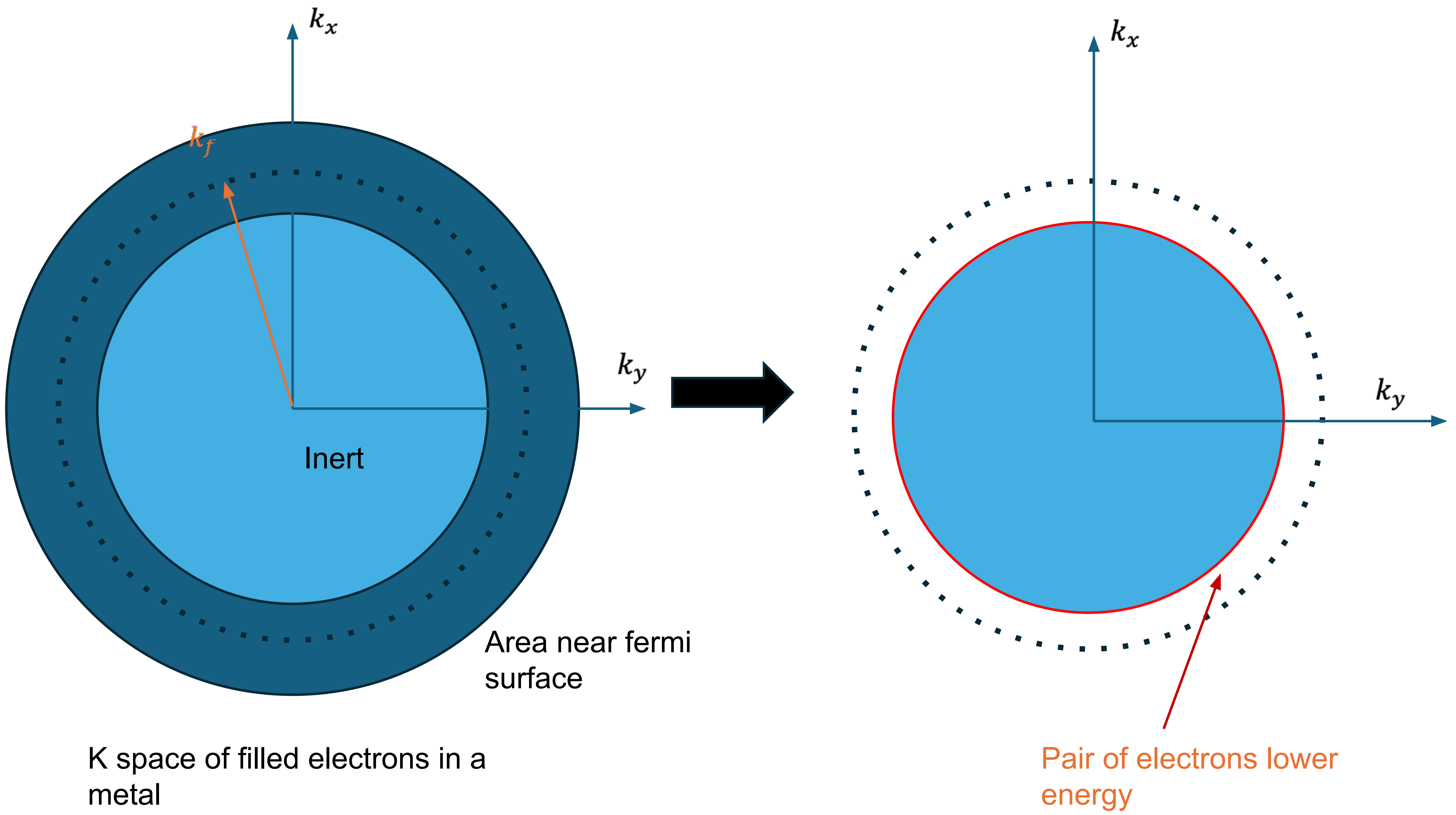
Assuming the density of states at the chemical potential to be $\mathrm{D}(\mu)$

$$E - \mu\bar{N} \equiv E_G \approx \mathrm{E}_{\mathrm{metal}} - \frac{1}{2}\mathrm{D}(\mu)\Delta^2$$

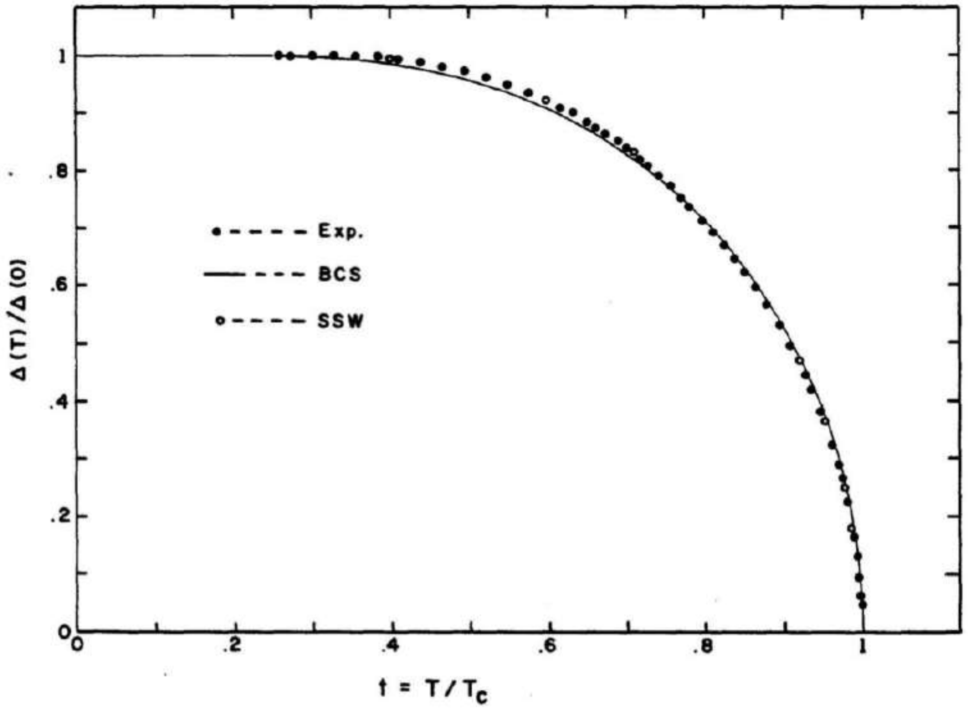
$$\Delta = 2\hbar\omega_D e^{-1/D(\mu)g}$$ Gap function

Ground state has lower energy for any small attractive interaction g!

This tells us that the ground state is gapped with respect to the fermi energy!


k_x
k_f
k_y
Inert
Area near fermi surface
k_x
k_y
K space of filled electrons in a metal
Pair of electrons lower energy

Does this work in practice?



Temperature dependence of the energy gap in Pb as determined by tunneling versus prediction of BCS theory

Deviations from the BCS theory are accounted for by numerical calculations at strong coupling by Swihart, Scalapino, and Wada (1965).

It doesn't just work. It works amazingly well!

From R. F. Gasparovic, B. N. Taylor, and R. E. Eck, Sol. State Comm. 4, 59 (1966).

Thank
you!